

ESAT

Maths 1

Number and units

Free lesson notes

Covers **M1.1 to M1.2, M2.1 to M2.6, M2.8 to M2.10, M2.14**

Format **Lesson, worked examples, self-check**

Price **Free**

About this lesson

Number questions on the ESAT are rarely about hard arithmetic. They test whether you can choose the clever route: prime factors instead of long division, standard form instead of a string of zeros, a quick estimate instead of an exact calculation, a unit conversion done in the right order. With no calculator and about ninety seconds a question, structure beats brute force.

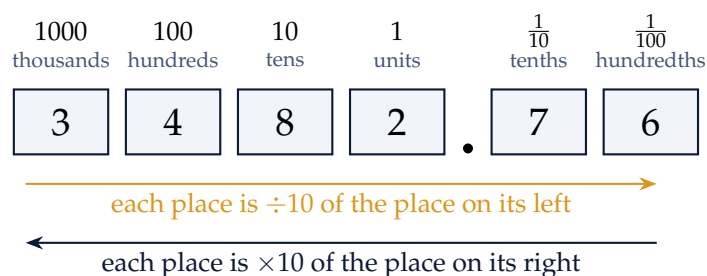
This lesson builds the subject from the ground up. We start with place value, ordering and the order of operations, then powers and roots. We study primes, factors, highest common factors (HCF) and lowest common multiples (LCM) with Venn diagrams; then fractions, decimals and recurring decimals; standard form; units and compound units (speed and density); applied speed problems including trains passing each other; counting methods (the multiplication principle and factorials, taught from scratch); and finally rounding and estimation. Indices and surds follow in the next lesson.

How to use it. Read each idea, then cover the worked example and try it yourself before reading the solution. Draw the diagram whenever there is one in the lesson: most of the cleverness in these questions is in the picture. Finish with the self-check, then the practice set. Specification codes: M1.1–1.2 (units), M2.1–2.6, M2.8–2.10 and M2.14 (number).

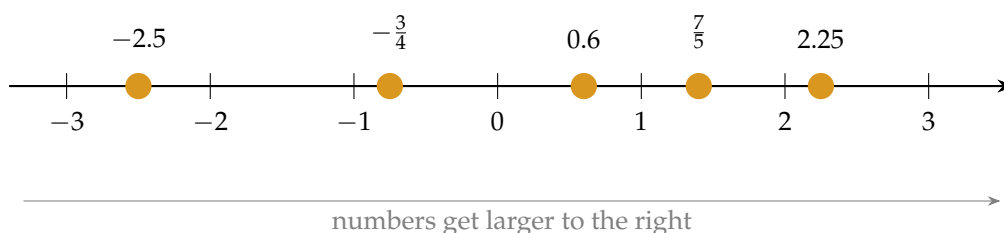
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1 Place value, ordering and the order of operations

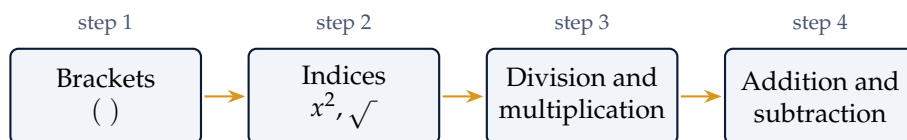
Place value. Every digit in a number has a value that depends on its position. In 3482.76 the 4 is worth 400 because it sits in the hundreds place, and the 7 is worth $\frac{7}{10}$ because it sits in the tenths place. Each place is worth ten times the place to its right, which is why multiplying by 10 moves every digit one place to the *left*, and dividing by 10 moves it one place to the right.



Ordering numbers. To compare decimals, line up the decimal points and compare place by place from the left, adding zeros so that the lengths match: 0.6, 0.58, 0.612 become 0.600, 0.580, 0.612, so the order is $0.58 < 0.6 < 0.612$. A longer decimal is not a larger one. For negative numbers, the further a number is to the left on the number line, the smaller it is: $-3 < -2.5 < -0.75$. To compare fractions, convert them to decimals or to a common denominator.



The order of operations. The expression $2 + 3 \times 4$ would have two possible answers unless everyone agrees on a convention. The agreed order is: brackets first, then indices (powers and roots), then division and multiplication, then addition and subtraction. Within the last two stages, work from left to right.



within the last two boxes, work from left to right

Worked example: order of operations

Evaluate $18 - 4 \times (5 - 2)^2 \div 6$.

Brackets: $5 - 2 = 3$. Indices: $3^2 = 9$. Now multiplication and division left to right: $4 \times 9 = 36$, then $36 \div 6 = 6$. Finally $18 - 6 = 12$.

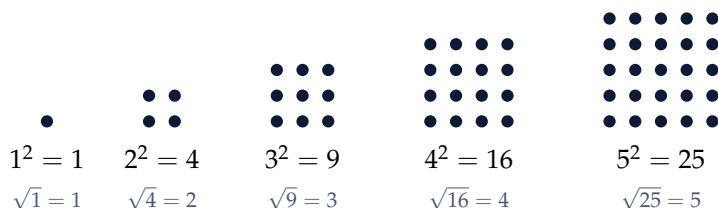
Inverse operations. Some questions run a calculation backwards: "I think of a number, multiply it by 3, subtract 4, and get 17." To undo a sequence, apply the *inverse* of each step in the *reverse* order (the last thing you put on is the first thing you take off): add 4, then divide by 3.

Worked example: working backwards

A number is multiplied by 3, then 4 is subtracted, giving 17. Find the number.

Undo the subtraction: $17 + 4 = 21$. Undo the multiplication: $21 \div 3 = 7$. Check: $7 \times 3 - 4 = 17$.

Squares, cubes and roots. A *square number* counts the dots in a square array: n^2 is n rows of n dots. A *cube number* $n^3 = n \times n \times n$ counts the unit blocks in a cube. The *square root* $\sqrt{}$ undoes squaring (it means the positive root), and the *cube root* $\sqrt[3]{}$ undoes cubing.



You should know the squares up to $15^2 = 225$ and the cubes 1, 8, 27, 64, 125, 216 (up to 6^3) and $10^3 = 1000$. To find a root of a decimal, use powers of ten: $0.0049 = 49 \times 10^{-4}$, so $\sqrt{0.0049} = 7 \times 10^{-2} = 0.07$; and $0.000216 = 216 \times 10^{-6}$, so $\sqrt[3]{0.000216} = 6 \times 10^{-2} = 0.06$. To locate a root that is not exact, sandwich it between neighbouring squares: $8^2 = 64 < 70 < 81 = 9^2$, so $\sqrt{70}$ lies between 8 and 9 (closer to 8, since 70 is nearer to 64).

Worked example: squares and roots

How many integers n (positive or negative) satisfy $50 < n^2 < 500$?

$7^2 = 49 < 50 < 64 = 8^2$ and $22^2 = 484 < 500 < 529 = 23^2$. For positive n we need $n = 8, 9, \dots, 22$: that is 15 values.

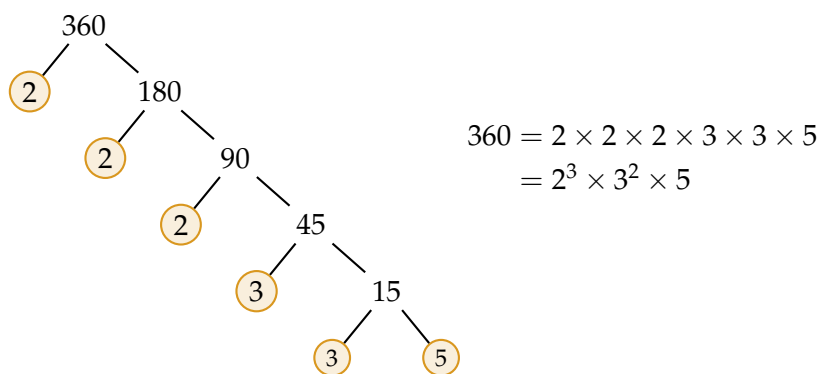
The squares of $-8, \dots, -22$ are the same, so there are 15 more. Total: 30.

Common mistakes

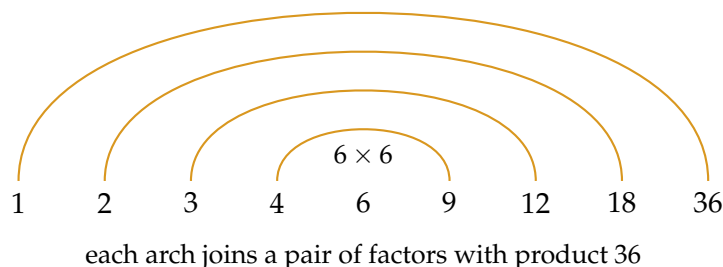
- Reading -3^2 as $+9$. The index applies to the 3 only, so $-3^2 = -9$, whereas $(-3)^2 = +9$.
- Doing $a - b \times c$ as $(a - b) \times c$. Multiplication comes first.
- Comparing 0.35 and 0.5 by length of decimal. Pad with zeros: $0.35 < 0.50$.

2 Primes, factors, HCF and LCM

A **factor** of a number divides it exactly, and a **multiple** is in its times table. A **prime** number has exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, ... (The number 1 is not prime, and 2 is the only even prime.) Every integer above 1 is a product of primes in exactly one way, its *prime factorisation*. A factor tree finds it: keep splitting until every branch ends in a prime.



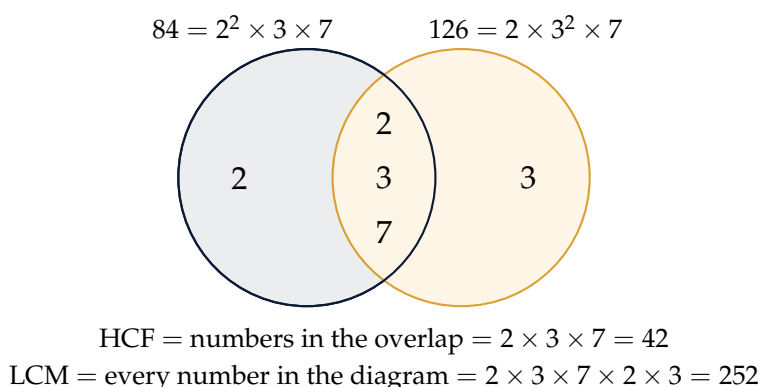
Counting factors. Factors of 36 come in pairs that multiply to 36, shown by the arches below. The middle factor 6 is paired with itself, because 36 is a perfect square.



There is a quick way to count all the factors from the prime factorisation. A factor of $360 = 2^3 \times 3^2 \times 5$ is built by choosing how many 2s to use (0, 1, 2 or 3: four choices), how many 3s (0, 1 or 2: three choices) and whether to use the 5 (0 or 1: two choices). The choices are independent, so the total is $4 \times 3 \times 2 = 24$. In general, if $N = p^a q^b r^c$ then N has $(a+1)(b+1)(c+1)$ factors. The “+1” is there because the power 0 is also allowed. It also follows that N is a perfect square exactly when every index is even (then $\sqrt{N}$ is a whole number, and the number of factors is odd).

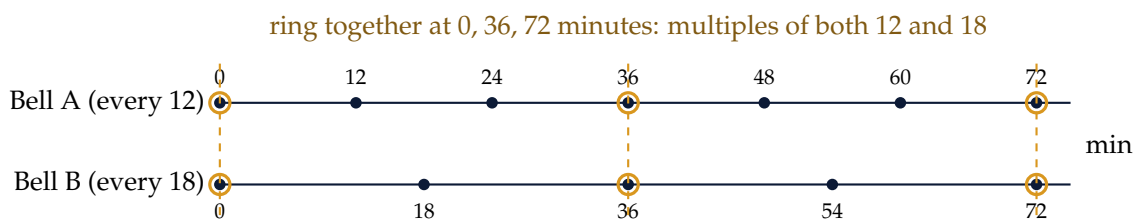
HCF and LCM. The *highest common factor* is the largest number dividing both numbers. The *lowest common multiple* is the smallest number that both divide. A Venn diagram of prime factors shows both at once. Write each number as a product of primes. Primes that appear in both go in the overlap; the rest go in the separate parts. Then

- HCF = the product of the numbers in the overlap;
- LCM = the product of *all* the numbers in the diagram.



In index form: for the HCF take each prime to the *lowest* power that appears in either number; for the LCM take the *highest*. The diagram also explains the useful fact $\text{HCF} \times \text{LCM} = \text{product of the two numbers}$. If h is the HCF, the two numbers are xh and yh (where x and y are what is left outside the overlap), and the LCM is xyh ; so $h \times xyh = (xh)(yh)$.

Why the LCM matters. Suppose bell A rings every 12 minutes and bell B every 18 minutes, both starting together. The times they ring together are the numbers in both times tables, and the first of them is the LCM.



Worked example: HCF and LCM

Find the HCF and LCM of 84 and 126.

$84 = 2^2 \times 3 \times 7$ and $126 = 2 \times 3^2 \times 7$. The overlap holds 2, 3, 7 so the HCF is 42. The other numbers are a 2 (from 84) and a 3 (from 126), so the LCM is $42 \times 2 \times 3 = 252$. Check: $42 \times 252 = 10584 = 84 \times 126$.

Worked example: tiling a rectangle

A floor is 72 cm by 108 cm. It is to be covered exactly with identical square tiles, as large as possible. What is the side of a tile and how many tiles are needed?

The side must divide both lengths, and be as large as possible, so it is the HCF. $72 = 2^3 \times 3^2$ and $108 = 2^2 \times 3^3$, so the HCF is $2^2 \times 3^2 = 36$ cm. There are $72 \div 36 = 2$ rows of $108 \div 36 = 3$ tiles: 6 tiles.

Worked example: a perfect square

What is the smallest positive integer n such that $1620n$ is a perfect square?

$1620 = 2^2 \times 3^4 \times 5$. A perfect square needs every index even. The indices of 2 and 3 are already even; the 5 has index 1, so $n = 5$. (Then $1620 \times 5 = 8100 = 90^2$.)

Common mistakes

- Taking the highest power for the HCF. The HCF uses the *lowest* powers; the LCM the highest.
- Forgetting that 1 and the number itself are factors when counting.
- Using the product $a \times b$ as the LCM. It is the LCM only when the numbers share no prime factor.

3 Fractions, decimals and percentages

Fractions. Equivalent fractions are found by multiplying or dividing top and bottom by the same number: $\frac{2}{3} = \frac{4}{6}$. To *add* or *subtract*, first write both with a common denominator, then add the tops only. To *multiply*, multiply tops and multiply bottoms. To *divide*, multiply by the reciprocal (flip the second fraction). The reason: $\frac{4}{9}$ fits into 1 exactly $\frac{9}{4}$ times, so dividing by $\frac{4}{9}$ is the same as multiplying by $\frac{9}{4}$.

Worked example: fraction arithmetic

Calculate $\frac{2}{3} + \frac{3}{4}$ and $\frac{2}{3} \div \frac{4}{9}$.

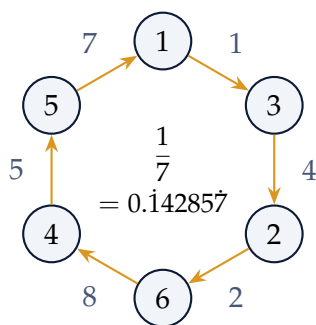
Common denominator 12: $\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$. For the division, $\frac{2}{3} \times \frac{9}{4} = \frac{18}{12} = \frac{3}{2}$.

Fractions, decimals and percentages are three ways of writing the same number. A percentage is “out of 100”: $35\% = \frac{35}{100} = \frac{7}{20} = 0.35$. To turn a fraction into a decimal, divide the top by the bottom. It is worth knowing the common ones by heart:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$
0.5	0.25	0.75	0.2	0.125	0.375	0.3	0.6	0.1
50%	25%	75%	20%	12.5%	37.5%	33.3%	66.6%	10%

Which fractions terminate? A decimal terminates when the fraction can be written with a denominator 10, 100, 1000, ... Since $10 = 2 \times 5$, that is possible exactly when the denominator (in lowest terms) has no prime factors other than 2 and 5: $\frac{3}{8} = 0.375$ terminates, but $\frac{1}{12} = \frac{1}{2^2 \times 3}$ does not, because of the 3.

Why decimals repeat. In long division the next digit depends only on the remainder. For $\frac{1}{7}$ the remainders go $1 \rightarrow 3 \rightarrow 2 \rightarrow 6 \rightarrow 4 \rightarrow 5 \rightarrow 1$, so after six steps we are back where we started and the digits repeat: $\frac{1}{7} = 0.\dot{1}4285\dot{7}$. Dividing by 7 can only give remainders 1 to 6 (or 0, when it terminates), so the cycle can never be longer than 6 digits.



circles: the remainder at each step
 digits outside: the next digit of the decimal
 the remainder 1 comes round again,
 so the digits repeat in a cycle of 6

From a recurring decimal to a fraction. Write the dots as above, and let x be the number. Multiply by a power of ten that moves the decimal point just past one whole repeating block; then subtract x so that the infinitely long repeating tails cancel. For $x = 0.\overline{45} = 0.4545\dots$ the block has two digits, so multiply by 100:

$$100x = 45.4545\dots, \quad x = 0.4545\dots \Rightarrow 99x = 45 \Rightarrow x = \frac{45}{99} = \frac{5}{11}.$$

If some digits come before the repeating part, as in $0.7\overline{2}$, use two multiplications so that both results have the same repeating tail.

Worked example: a mixed recurring decimal

Write $0.7\overline{2} = 0.72222\dots$ as a fraction.

Let $x = 0.7222\dots$. Then $10x = 7.222\dots$ and $100x = 72.222\dots$ (the same tail). Subtracting, $90x = 72.222\dots - 7.222\dots = 65$, so $x = \frac{65}{90} = \frac{13}{18}$.

Worked example: comparing

Which is the largest: $\frac{7}{11}$, 0.636 , 63.7% , $\frac{5}{8}$, $0.6\dot{3}$?

Write all as decimals, to four places: 0.6364 , 0.6360 , 0.6370 , 0.6250 , 0.6333 . The largest is 63.7% .

Common mistakes

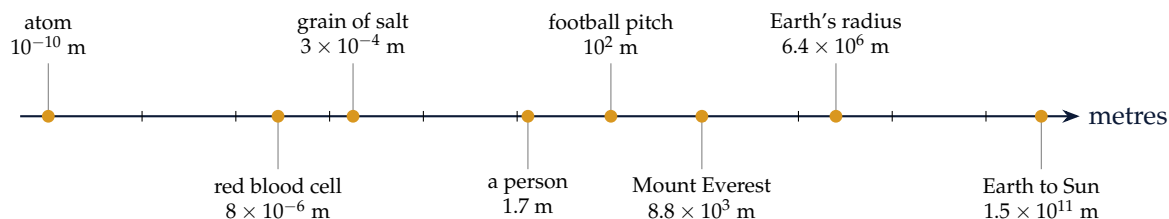
- Writing $0.\overline{45}$ as $\frac{45}{100}$. Two repeating digits means a denominator of 99 (before simplifying).
- Adding fractions by adding the tops *and* the bottoms: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.
- Comparing decimals by length: 0.62 is smaller than $0.6\overline{2} = 0.6222\dots$

4 Standard form

Very large and very small numbers are awkward to write with lots of zeros. **Standard form** writes a number as

$$a \times 10^n, \quad 1 \leq a < 10, \quad n \text{ an integer.}$$

The power 10^n with $n > 0$ is 1 followed by n zeros ($10^3 = 1000$); with $n < 0$ it is $1 \div 10^{|n|}$ ($10^{-2} = 0.01$). To convert, move the decimal point until the number is between 1 and 10 and count the moves: $4\,500\,000 = 4.5 \times 10^6$ (six places left) and $0.00032 = 3.2 \times 10^{-4}$ (four places right).



Multiplying and dividing. Treat the number parts and the powers of ten separately. $10^a \times 10^b = 10^{a+b}$ because 10^a is a tens multiplied together, and 10^b is b more; and $10^a \div 10^b = 10^{a-b}$. Afterwards, check that the answer is still in standard form. **Adding and subtracting:** first write both numbers with the same power of ten.

Worked example: multiplying

Calculate $(3.6 \times 10^5) \times (5 \times 10^{-8})$, giving the answer in standard form.
 $3.6 \times 5 = 18$ and $10^5 \times 10^{-8} = 10^{-3}$, so the product is 18×10^{-3} . But $18 \geq 10$, so this is not yet standard form:
 $18 \times 10^{-3} = 1.8 \times 10^{-2}$.

Worked example: dividing

Calculate $\frac{6 \times 10^{-3}}{1.5 \times 10^{-6}}$.
 $6 \div 1.5 = 4$. For the powers, $10^{-3} \div 10^{-6} = 10^{-3-(-6)} = 10^3$. The answer is 4×10^3 .

Worked example: adding

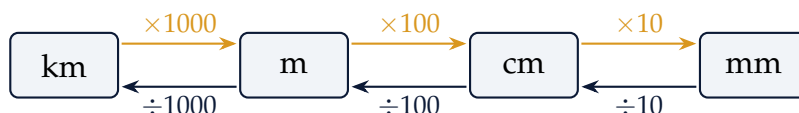
Calculate $3.2 \times 10^4 + 5 \times 10^3$ in standard form.
Write both with 10^3 : $32 \times 10^3 + 5 \times 10^3 = 37 \times 10^3 = 3.7 \times 10^4$.

Common mistakes

- Stopping at 18×10^{-3} . The number part must be at least 1 and less than 10.
- Adding the indices when dividing. Dividing subtracts them, and subtracting a negative index adds.
- Adding the number parts of $3.2 \times 10^4 + 5 \times 10^3$ without matching the powers.

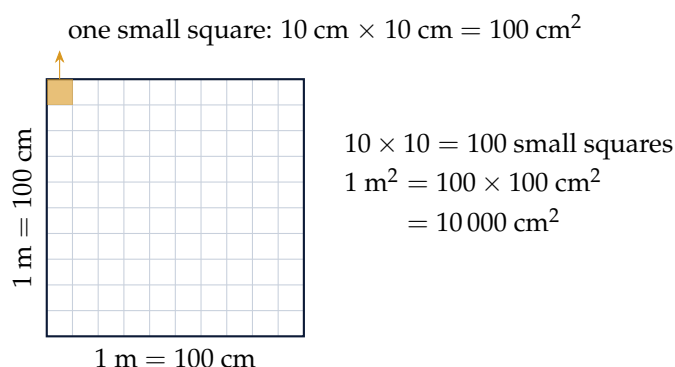
5 Units and compound units

A *compound unit* combines two units: speed in m/s (“metres *per* second”), density in g/cm³, pressure in N/m². The prefixes you need are *kilo* ($\times 1000$), *centi* ($\div 100$) and *milli* ($\div 1000$): 1 km = 1000 m, 1 m = 100 cm = 1000 mm, 1 kg = 1000 g.



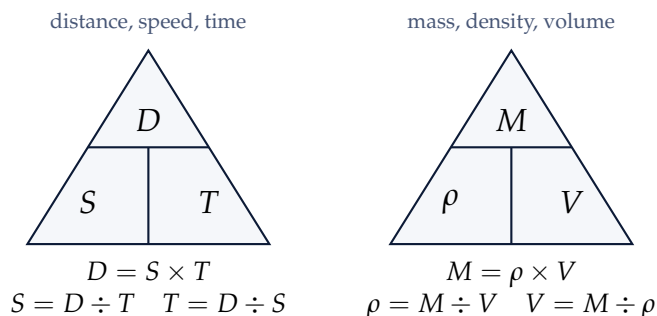
Going to a *smaller* unit needs *more* of them, so you multiply; going to a larger unit, you divide. A good habit is to ask: “should my answer be a bigger or smaller number?” before calculating.

Areas and volumes. The conversion factor squares for an area and cubes for a volume. Since 1 m = 100 cm, a square metre is 100 cm by 100 cm. Tiling it with small 10 cm squares (each 100 cm²) takes $10 \times 10 = 100$ of them, so $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$.



In the same way $1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$. Also useful: $1 \text{ cm}^3 = 1 \text{ ml}$ and 1 litre = 1000 cm³.

Compound units: convert the top and the bottom separately. For speed, $1 \text{ km/h} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{1}{3.6} \text{ m/s}$, so to change km/h into m/s divide by 3.6, and to change m/s into km/h multiply by 3.6. Density is mass per unit volume, and the three quantities are linked by the triangles below: cover the one you want and read off the formula.



Worked example: speed

Convert 72 km/h to m/s.

72 km = 72 000 m and 1 h = 3600 s, so the speed is $\frac{72\,000}{3600} = 20$ m/s. (Or $72 \div 3.6 = 20$.)

Worked example: density

Convert 2.7 g/cm³ to kg/m³.

Top: 1 g = $\frac{1}{1000}$ kg. Bottom: 1 cm³ = $\frac{1}{10^6}$ m³, so 1 g/cm³ = $\frac{10^{-3}}{10^{-6}} = 1000$ kg/m³. Hence 2.7 g/cm³ = 2700 kg/m³.

Worked example: volume in litres

A tank is 2.4 m long, 1.5 m wide and 0.5 m deep. How many litres does it hold?

Volume = $2.4 \times 1.5 \times 0.5 = 1.8$ m³ = $1.8 \times 1\,000\,000$ cm³ = 1 800 000 cm³. Dividing by 1000 gives 1800 litres. (A cube of side 1 m holds 1000 litres.)

Common mistakes

- Multiplying by 100 to go from m² to cm². It is $\times 100^2$.
- Converting the top of a compound unit but forgetting the bottom.
- Writing 2 h 15 min as 2.15 h. It is $2\frac{15}{60} = 2.25$ h.

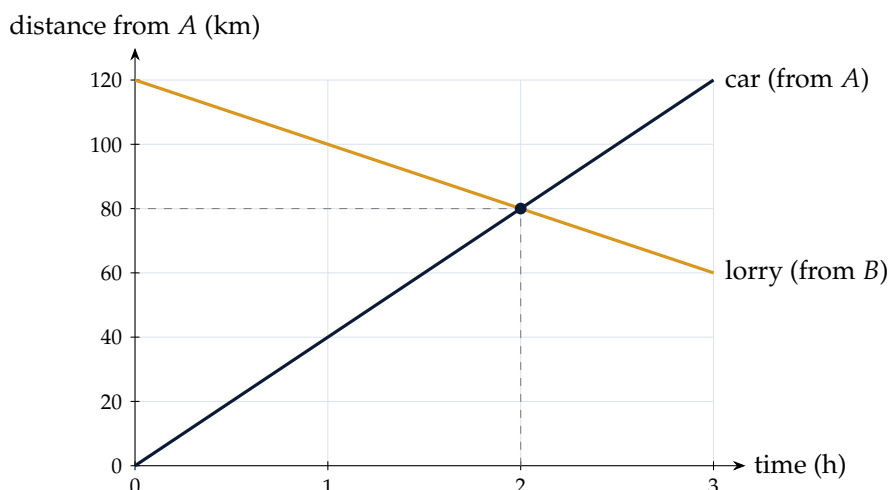
6 Speed, distance and time: averages, graphs and trains

Average speed. Speed is distance divided by time. The *average* speed for a whole journey is always

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}.$$

It is *not* the average of the speeds. If you drive 60 km at 30 km/h (taking 2 h) and then 60 km at 60 km/h (taking 1 h), the journey is 120 km in 3 h, an average of 40 km/h, not 45. You spend longer at the slower speed.

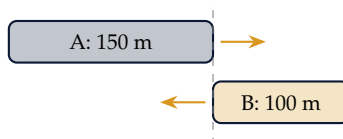
Distance–time graphs. On a graph of distance against time, the *gradient* is the speed: a steeper line means a faster journey, a horizontal line means stopped. Where two lines cross, the two travellers are at the same place at the same time. The picture shows a car leaving A at 40 km/h and a lorry leaving B, 120 km away, at 20 km/h towards A.



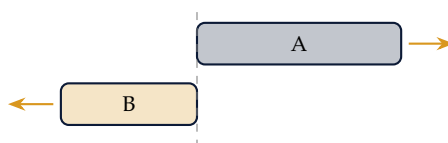
Each car's distance from A is $40t$ and $120 - 20t$, which are equal when $60t = 120$: $t = 2$ hours, at 80 km from A . The shortcut: because they are moving *towards* each other, they close the 120 km gap at $40 + 20 = 60$ km/h, so they meet after $120 \div 60 = 2$ h. If they moved in the *same* direction the gap would close at the *difference* of the speeds.

Trains passing. The key idea is *relative motion*. Look at the pair of trains from the point of view of one of them: it is as if that train is stationary, and the other moves at the closing speed (the sum of the speeds if they approach, the difference if one overtakes the other). Passing starts when the two *fronts* meet and is complete when the two *rears* have cleared one another. In that time the relative distance travelled is exactly the sum of the two lengths.

Start: the fronts meet



End: the rears have just cleared



distance travelled relative to each other = $150 + 100 = 250$ m

Worked example: trains passing head-on

Train A is 180 m long and travels at 25 m/s. Train B is 120 m long and travels at 15 m/s in the opposite direction on a parallel track. How long do they take to pass each other completely?
Closing speed $25 + 15 = 40$ m/s. Distance to clear: $180 + 120 = 300$ m. Time = $300 \div 40 = 7.5$ s.

Worked example: overtaking

A train 200 m long travelling at 30 m/s overtakes a train 100 m long travelling at 20 m/s in the same direction. How long does the overtaking take?
Overtaking begins when the front of the fast train is level with the *rear* of the slow train, and ends when the rear of the fast train is level with the *front* of the slow train. The relative distance is again the sum of the lengths, 300 m, but now the closing speed is the *difference* of the speeds, $30 - 20 = 10$ m/s. Time = $300 \div 10 = 30$ s.

Worked example: a train and a pole

A train takes 8 s to pass a signal pole and 20 s to cross a bridge 240 m long. How long is the train?
Passing the pole means covering its own length L in 8 s, so its speed is $L/8$. Crossing the bridge means covering

$L + 240$ in 20 s, so $\frac{L + 240}{20} = \frac{L}{8}$. Then $8L + 1920 = 20L$, giving $L = 160$ m (and the speed is 20 m/s).

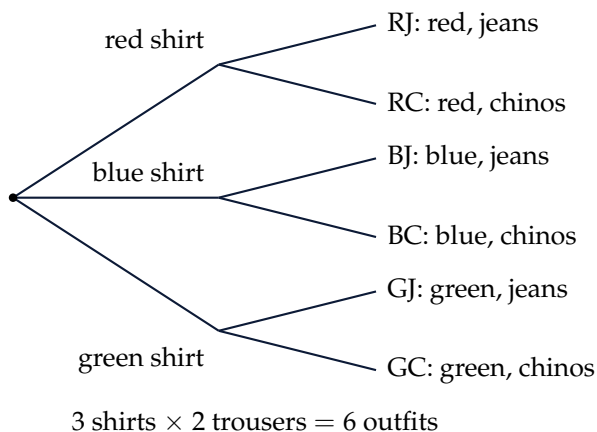
Common mistakes

- Taking an average of speeds instead of total distance over total time.
- Using the sum of speeds when two trains travel in the *same* direction (use the difference).
- Forgetting that the distance for passing is the sum of the two lengths, not the length of one train.
- Mixing units: if the speed is in km/h and the length in metres, convert first.

7 Systematic listing and counting methods

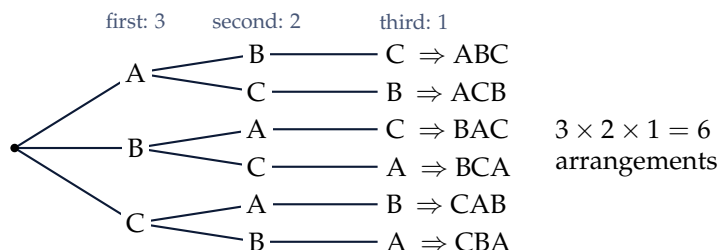
Listing systematically. To count possibilities without missing any or repeating any, list them in a fixed order (like alphabetical order). For example, the ways to write 6 as a sum of two positive whole numbers, ignoring order, are $1 + 5$, $2 + 4$, $3 + 3$: three ways. Listing is reliable but slow, so for bigger problems we need a counting rule.

The multiplication principle. Suppose an outfit is a shirt and a pair of trousers, with 3 shirts and 2 pairs of trousers. For each shirt there are the same number, 2, of ways to finish the outfit. The tree shows it: 3 branches, each splitting into 2, giving $3 \times 2 = 6$ outfits.



In general, if a task is done in stages, and there are a ways to do the first stage, then b ways to do the second whatever happened first, then c ways for the third, the total number of ways is $a \times b \times c$. Notice that we multiply whenever choices are made *one after another* and every combination is allowed, and that the number of choices at a later stage may be smaller because earlier choices have used things up.

Arrangements and factorials. In how many ways can the three letters A, B, C be arranged in a row? The first position has 3 choices, the second has 2 (one letter has been used) and the third has just 1. By the multiplication principle there are $3 \times 2 \times 1 = 6$ arrangements, and the tree lists all of them.



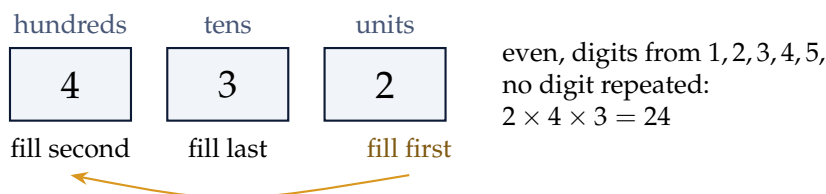
The product $3 \times 2 \times 1$ is written $3!$, read “three factorial”. In general

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1,$$

the number of ways to arrange n different objects in a row. So $4! = 24$, $5! = 120$ and $6! = 720$. Factorials grow quickly, so it is worth being able to write $n!$ as a product and cancel, rather than compute huge numbers: $\frac{7!}{5!} = 7 \times 6 = 42$. If some

objects are *identical*, swapping them does not give a new arrangement, so divide by the number of ways to rearrange the identical ones: the letters AAB can be arranged in $\frac{3!}{2!} = 3$ ways (AAB, ABA, BAA).

Restrictions: deal with the restricted place first. Count the digit with a rule on it first, then fill in the others. For even three-digit numbers from the digits 1, 2, 3, 4, 5 without repeats, the units digit must be 2 or 4 (2 choices); the hundreds digit can be any of the remaining 4; the tens digit any of the remaining 3.



Worked example: a zero that cannot lead

How many odd three-digit numbers can be made from the digits 0, 1, 2, 3, 4 if no digit is used twice? (A number cannot start with 0.)

Two places have rules: the units digit (odd: 1 or 3) and the hundreds digit (not 0). Do the units digit first: 2 choices. The hundreds digit cannot be 0 or the digit just used, so from 4 remaining digits there are 3 choices. The tens digit is any of the 3 left. Total $2 \times 3 \times 3 = 18$.

Worked example: keeping two people together

Five friends A, B, C, D, E sit in a row. In how many ways can they sit if A and B must be next to each other?

Glue A and B into one block. Then there are 4 things to arrange (the block, C, D, E), in $4! = 24$ ways. Inside the block, A and B can swap, giving 2 ways. Total $24 \times 2 = 48$.

Worked example: counting by subtraction

How many 3-letter arrangements from A, B, C, D, E (no repeats) do *not* contain the letter A ?

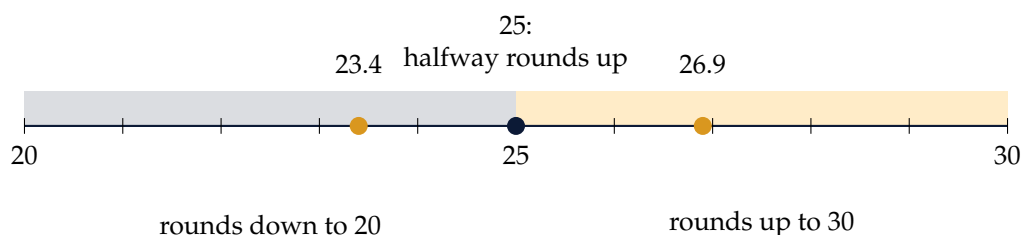
All arrangements: $5 \times 4 \times 3 = 60$. Those without A use only B, C, D, E : $4 \times 3 \times 2 = 24$. (Those containing A number $60 - 24 = 36$.) Counting the "opposite" and subtracting from the total is often quicker than counting what you want directly.

Common mistakes

- Adding the numbers of choices when the choices are made one after another. Multiply.
- Forgetting that a choice has used something up: the second stage often has one fewer choice.
- Starting with the unrestricted places, then having to untangle the restriction. Do the restricted place first.
- Computing $n!$ in full when it cancels: write $\frac{7!}{5!} = 7 \times 6$.

8 Rounding and estimation

Rounding. To round to a given place, look at the digit to the right. If it is 5 or more, round up; otherwise round down. Numbers exactly half-way round up. The number line shows why: everything from 20 up to (but not including) 25 is closer to 20.



To round to *significant figures* (s.f.), count from the first non-zero digit: 0.004706 to 2 s.f. is 0.0047 and 48 917 to 2 s.f. is 49 000 (the zeros keep the place value). To round to *decimal places*, count digits after the point.

Estimating. Round every number to one significant figure, then calculate with the easy numbers. The result will not be exact, but it is usually near enough to rule out options that are wrong by a factor of 10 or by a misplaced decimal point, which are the most common slips. Remember that dividing by a number below 1 makes the answer *bigger*.

Worked example: estimating a calculation

Estimate $\frac{4.02 \times 19.8}{0.51}$.

Round to one significant figure: $\frac{4 \times 20}{0.5} = \frac{80}{0.5} = 160$. (The exact value is about 156.)

Worked example: size of an answer

Estimate $\frac{48.9 \times 0.0204}{0.0097}$.

$48.9 \approx 50$, $0.0204 \approx 0.02$ and $0.0097 \approx 0.01$. Then $\frac{50 \times 0.02}{0.01} = \frac{1}{0.01} = 100$. (Exact: about 102.8.)

Worked example: how many seconds in a year?

Estimate the number of seconds in a year in standard form.

$365 \times 24 \times 3600 \approx 4 \times 10^2 \times 2.5 \times 10^1 \times 3.6 \times 10^3 \approx 4 \times 2.5 \times 3.6 \times 10^6 = 36 \times 10^6 \approx 3 \times 10^7$. (A handy fact: a year is about $\pi \times 10^7$ seconds.)

Exam tip

Use estimation as a *sanity check on every answer*: is the answer plausible in size and in units? A speed of 500 km/h for a cyclist or a mass of 0.02 kg for a car are impossible, whatever your arithmetic said.

Common mistakes

- Rounding 0.0097 as 0.1: count the zeros after the decimal point carefully, $0.0097 \approx 0.01$.
- Rounding intermediate values too early and then rounding again: estimate once, at the start.
- Forgetting that dividing by a number less than 1 increases the answer.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Convert 90 km/h to m/s.
2. Find the LCM and HCF of 18 and 30.
3. How many factors does 200 have?
4. Write $0.\dot{8}\dot{1}$ as a fraction in its lowest terms.
5. Write $(2 \times 10^4)^3$ in standard form.
6. Two trains, 100 m and 140 m long, approach each other at 12 m/s and 18 m/s. How long do they take to pass?
7. How many different arrangements are there of the letters of the word MATH, and how many start with a vowel?
8. Estimate $\frac{29.7 \times 0.52}{0.99}$.

Answers to the self-check

1. $90 \div 3.6 = 25 \text{ m/s}$.
2. $18 = 2 \times 3^2$, $30 = 2 \times 3 \times 5$. $\text{HCF} = 2 \times 3 = 6$, $\text{LCM} = 2 \times 3^2 \times 5 = 90$ (check: $6 \times 90 = 540 = 18 \times 30$).
3. $200 = 2^3 \times 5^2$, so $(3 + 1)(2 + 1) = 12$.
4. $99x = 81$, so $x = \frac{81}{99} = \frac{9}{11}$.
5. $2^3 \times 10^{12} = 8 \times 10^{12}$.
6. Closing speed 30 m/s , distance 240 m , so 8 s .
7. $4! = 24$ in total; with A first there are $3! = 6$.
8. $\frac{30 \times 0.5}{1} = 15$.

Next step

Heurisco's *ESAT Maths 1: Number and Units Practice Set* has twenty original exam-style questions on this lesson, with diagrams, in order of difficulty, with a full worked solution for each.