

ESAT

Maths 1

Equations, inequalities and sequences

Free lesson notes

Covers

M4.15 to M4.19

Format

Lesson, worked examples, self-check

Price

Free

About this lesson

Most ESAT algebra comes down to one skill: turning words or structure into an equation and solving it cleanly. This lesson covers linear equations (including fractions), simultaneous equations (linear and linear-quadratic), quadratic equations by three methods, linear inequalities, and sequences, including n th terms of linear and quadratic sequences.

How to use it. Try each worked example first. Then do the self-check, then the practice set. In the exam, always *check your solution* in the original equation: it takes ten seconds and catches most slips.

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1 Linear equations and forming equations

Do the same thing to both sides. Expand brackets first, collect the x terms on one side and the numbers on the other. To clear fractions, multiply every term by the lowest common denominator.

To form an equation from a problem, define a letter for the unknown, write each piece of information as an expression, and join them with an equals sign.

Worked example: fractions

Solve $\frac{x+1}{2} - \frac{x-2}{3} = 2$.

Multiply every term by 6: $3(x+1) - 2(x-2) = 12$.

Expand: $3x + 3 - 2x + 4 = 12$, so $x + 7 = 12$ and $x = 5$.

Check: $\frac{6}{2} - \frac{3}{3} = 3 - 1 = 2$. ✓

Worked example: forming an equation

A father is 4 times as old as his son. In 6 years' time he will be 3 times as old. How old is the father now?

Let the son be s . Then $4s + 6 = 3(s + 6)$, so $4s + 6 = 3s + 18$ and $s = 12$.

The father is $4 \times 12 = 48$.

Common mistakes

- Forgetting to multiply *every* term by the common denominator, including whole numbers.
- Sign errors when subtracting a bracket: $-(x - 2) = -x + 2$.

2 Simultaneous equations

Two unknowns need two equations.

- **Elimination:** multiply the equations so one letter has equal coefficients, then add or subtract.
- **Substitution:** make one letter the subject of the simpler equation and substitute into the other. Always use this for a linear equation with a quadratic one, which gives a quadratic with up to two solutions.

Give answers as *pairs* and match each x with its own y by substituting back into the linear equation.

Worked example: elimination

Solve $2x + 3y = 7$ and $5x - 2y = -11$.

Multiply the first by 2 and the second by 3: $4x + 6y = 14$ and $15x - 6y = -33$.

Add: $19x = -19$, so $x = -1$. Then $3y = 7 + 2 = 9$, so $y = 3$.

Worked example: linear and quadratic

Solve $y = x + 2$ and $y = x^2 - 4$.
 Substitute: $x + 2 = x^2 - 4$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$.
 $x = 3 \Rightarrow y = 5$; $x = -2 \Rightarrow y = 0$. The solutions are $(3, 5)$ and $(-2, 0)$.

Common mistakes

- Giving only the x -values of a linear-quadratic system, or pairing them with the wrong y .
- Subtracting equations with a sign error. Write the subtraction out term by term.

3 Quadratic equations

Get everything on one side so the equation reads $ax^2 + bx + c = 0$, then choose a method.

- **Factorise** when you can: $2x^2 - 5x - 3 = (2x + 1)(x - 3)$, so $x = -\frac{1}{2}$ or $x = 3$.
- **Complete the square:** $x^2 + bx = (x + \frac{b}{2})^2 - \frac{b^2}{4}$. It shows the exact roots as surds.
- **The formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

For roots α and β : $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$, so you can find sums and products without solving.

Worked example: completing the square

Solve $x^2 - 6x + 2 = 0$ exactly.
 $(x - 3)^2 - 9 + 2 = 0$, so $(x - 3)^2 = 7$ and $x = 3 \pm \sqrt{7}$.

Worked example: the formula

Solve $3x^2 + 4x - 2 = 0$.

$$x = \frac{-4 \pm \sqrt{16 + 24}}{6} = \frac{-4 \pm \sqrt{40}}{6} = \frac{-4 \pm 2\sqrt{10}}{6} = \frac{-2 \pm \sqrt{10}}{3}.$$

Common mistakes

- Using $-4ac$ wrongly: when c is negative, $-4ac$ is *positive*, so the discriminant is $16 + 24 = 40$ above.
- Dividing only one term by $2a$: the whole numerator is divided.
- Cancelling the $\pm$ incorrectly: $\frac{-4 \pm 2\sqrt{10}}{6}$ simplifies by dividing *both* terms by 2.

4 Linear inequalities

Solve an inequality like an equation, with one rule: *multiplying or dividing by a negative number reverses the inequality sign*. Answers can be shown on a number line or written like $-1 < x \leq 5$. For integer solutions, list or count carefully, watching which ends are included.

Worked example: reversing the sign

Solve $7 - 3x \geq 19$.
 $-3x \geq 12$. Divide by -3 and reverse the sign: $x \leq -4$.

Worked example: counting integers

How many integers satisfy $-3 < 2x - 1 \leq 9$?
 Add 1: $-2 < 2x \leq 10$. Divide by 2: $-1 < x \leq 5$.
 The integers are 0, 1, 2, 3, 4, 5: that is 6.

Common mistakes

- Forgetting to reverse the sign when dividing by a negative.
- Counting -1 as a solution when $x > -1$ is strict.

5 Sequences

A sequence can be given by a *term-to-term rule* or by its *n th term*.

- **Linear** (constant first difference d): $u_n = dn + (u_1 - d)$. For 7, 11, 15, 19: $d = 4$, so $u_n = 4n + 3$.
- **Quadratic** (constant *second* difference): $u_n = an^2 + bn + c$ where $a = \frac{1}{2} \times$ second difference. Subtract an^2 from the terms to find the linear part.

To test whether a number is in a sequence, set u_n equal to it and check that n is a positive integer.

Worked example: a quadratic n th term

Find the n th term of 2, 9, 22, 41, ...

First differences: 7, 13, 19. Second differences: 6, 6, so $a = 3$.

Subtract $3n^2$ (3, 12, 27, 48) from the terms: $-1, -3, -5, -7$, which is $-2n + 1$.

So $u_n = 3n^2 - 2n + 1$. Check $n = 3$: $27 - 6 + 1 = 22$. ✓

Worked example: arithmetic sequence with an unknown

The terms k , $2k + 1$ and $5k - 4$ are consecutive terms of an arithmetic sequence. Find k and the 10th term.

Equal differences: $(2k + 1) - k = (5k - 4) - (2k + 1)$, so $k + 1 = 3k - 5$ and $k = 3$.

The terms are 3, 7, 11 with $d = 4$. The 10th term is $3 + 9 \times 4 = 39$.

Common mistakes

- Using the first difference as the coefficient of n^2 . It is *half* the second difference.
- Forgetting that n must be a whole number: a value like $n = 6.6$ means the number is not in the sequence.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Solve $\frac{2x - 1}{3} = \frac{x + 4}{2}$.
2. Solve $x + y = 9$ and $3x - y = 7$.
3. Solve $x^2 - 4x - 5 = 0$.
4. Solve $5 - 2x < 11$.
5. Find the n th term of 5, 8, 11, 14, ...
6. The n th term of a sequence is $n^2 + 3n$. Find the 6th term.

Answers to the self-check

1. $2(2x - 1) = 3(x + 4)$, so $4x - 2 = 3x + 12$ and $x = 14$.
2. Add: $4x = 16$, so $x = 4$ and $y = 5$.
3. $(x - 5)(x + 1) = 0$, so $x = 5$ or $x = -1$.
4. $-2x < 6$, so $x > -3$.
5. $d = 3$, so $u_n = 3n + 2$.
6. $36 + 18 = 54$.

Next step

Heurisco's *ESAT Maths 1: Equations, Inequalities and Sequences Practice Set* has twenty original exam-style questions on this lesson, in order of difficulty, with a full worked solution for each.