

ESAT

Maths 1

Graphs

Free lesson notes

Covers

M4.9 to M4.14

Format

Lesson, worked examples, self-check

Price

Free

About this lesson

Graph questions combine algebra and geometry, and nearly every one is solved by finding a small number of key facts: where the graph crosses the axes, its gradient, and its turning point. This lesson covers coordinates, straight lines (including parallel and perpendicular), quadratic graphs, sketching standard curves, and reading distance-time and speed-time graphs.

How to use it. Try each worked example first. Then do the self-check, then the practice set. Sketching quickly on the exam paper, labelling intercepts, is a habit worth building.

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1 Coordinates, gradient and distance

For points $A(x_1, y_1)$ and $B(x_2, y_2)$:

Three formulae

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance formula is Pythagoras applied to the horizontal and vertical differences.

Worked example: midpoint and distance

Find the midpoint and length of AB for $A(-2, 5)$ and $B(6, -1)$.

Midpoint: $\left(\frac{-2 + 6}{2}, \frac{5 + (-1)}{2} \right) = (2, 2)$.

Differences: $\Delta x = 8$, $\Delta y = -6$, so $AB = \sqrt{64 + 36} = 10$.

Common mistakes

- Subtracting coordinates in different orders for x and y in the gradient: use the same order top and bottom.
- Forgetting the square root in the distance formula.

2 Straight lines

A line can be written $y = mx + c$ (m the gradient, c the y -intercept) or $ax + by = c$.

- **Parallel** lines have the same gradient.
- **Perpendicular** lines have gradients with product -1 : $m_1 m_2 = -1$, so the gradient is the *negative reciprocal*.
- To find intercepts: set $x = 0$ for the y -intercept and $y = 0$ for the x -intercept.
- Through (x_1, y_1) with gradient m : $y - y_1 = m(x - x_1)$.

Worked example: parallel line

Find the line through $(3, 1)$ parallel to $2x + 3y = 6$.

Rearrange: $y = -\frac{2}{3}x + 2$, so the gradient is $-\frac{2}{3}$.

The parallel line has $2x + 3y = k$. At $(3, 1)$: $k = 6 + 3 = 9$. So $2x + 3y = 9$.

Worked example: perpendicular line and intercept

A line passes through $(0, 4)$ and is perpendicular to $y = \frac{1}{2}x + 1$. Where does it cross the x -axis?

Gradient $= -2$, so $y = -2x + 4$. Set $y = 0$: $x = 2$. The point is $(2, 0)$.

Common mistakes

- Taking the *reciprocal* but not changing the sign: the perpendicular to gradient $\frac{1}{2}$ is -2 , not 2 .
- Swapping the intercepts: the x -intercept sets $y = 0$.

3 Quadratic graphs

The graph of $y = ax^2 + bx + c$ is a parabola: $\cup$ -shaped if $a > 0$, $\cap$ -shaped if $a < 0$.

- Roots (the x -intercepts): factorise or use the formula.
- The turning point is on the line of symmetry $x = -\frac{b}{2a}$, halfway between the roots.
- Completing the square, $y = a(x + p)^2 + q$, gives the turning point $(-p, q)$ directly.

Worked example: turning point

Find the minimum value of $y = x^2 - 6x + 5$.
 $y = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$. The minimum value is -4 , at $x = 3$.

Worked example: from the roots

A parabola crosses the x -axis at $x = 1$ and $x = 3$ and the y -axis at $y = -3$. Find its maximum value.
 $y = a(x - 1)(x - 3)$ and $-3 = a(-1)(-3) = 3a$, so $a = -1$.
 The line of symmetry is $x = 2$, where $y = -(1)(-1) = 1$. The maximum value is 1 .

Common mistakes

- Reading the minimum value as the x -coordinate of the turning point. The question may ask for the value of y .
- Forgetting that $a < 0$ means a maximum.

4 Sketching standard graphs

Recognise these shapes and their key features:

- **Cubic** $y = a(x - p)(x - q)(x - r)$: crosses the axis at p, q, r ; for $a > 0$ it rises on the right and falls on the left.
- **Reciprocal** $y = \frac{k}{x}$: two branches, never touching either axis (asymptotes $x = 0, y = 0$). $k > 0$: branches in quadrants 1 and 3.
- **Exponential** $y = ab^x$: passes through $(0, a)$, grows if $b > 1$, decays if $0 < b < 1$, never reaches the x -axis.
- **Trigonometric** $y = a \sin(bx)$: amplitude $|a|$ and period $\frac{360^\circ}{b}$.

To match an equation to a graph, check the intercepts and the end behaviour first.

Worked example: exponential

The curve $y = ab^x$ passes through $(0, 3)$ and $(2, 12)$. Find y when $x = 3$.
 At $x = 0$: $a = 3$. At $x = 2$: $3b^2 = 12$, so $b = 2$ (taking $b > 0$).
 At $x = 3$: $y = 3 \times 2^3 = 24$.

Worked example: sine curve

The graph of $y = a \sin(bx)$ has maximum value 3 and period 120° . Find $a + b$.
 Amplitude: $a = 3$. Period: $\frac{360}{b} = 120$, so $b = 3$. Then $a + b = 6$.

Common mistakes

- Reading $y = ab^x$ as passing through $(0, b)$. The y -intercept is a , because $b^0 = 1$.
- Using the period as $b \times 360^\circ$ rather than $360^\circ \div b$.

5 Interpreting graphs

Two facts cover most kinematic graphs:

Gradient and area

- On a **distance-time** graph, the gradient is the *speed*. A flat section means stationary.
- On a **speed-time** graph, the gradient is the *acceleration*, and the *area under the graph* is the distance travelled.

Split areas into triangles, rectangles and trapezia.

Worked example: area under a speed-time graph

A car accelerates uniformly from rest to 12 m/s in 4 s, holds that speed for 6 s, then slows uniformly to rest in 5 s. How far does it travel?

$$\text{Area} = \frac{1}{2}(4)(12) + 6(12) + \frac{1}{2}(5)(12) = 24 + 72 + 30 = 126 \text{ m.}$$

Worked example: using the area

A train decelerates uniformly from 30 m/s to rest over 450 m. How long does this take?

$$\text{Distance} = \text{area} = \frac{1}{2} \times 30 \times t = 450, \text{ so } t = 30 \text{ s.}$$

Common mistakes

- Using speed $\times$ time for an accelerating section. Only constant speed gives a rectangle.
- Reading the gradient of a speed-time graph as speed. It is acceleration.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Find the gradient of the line through (2, 9) and (5, 0).
2. Find the equation of the line through (0, 1) perpendicular to $y = 3x - 2$.
3. Find the minimum value of $y = x^2 + 4x + 7$.
4. Find the distance between (1, -2) and (7, 6).
5. A line has equation $2x - 5y = 20$. Where does it cross the axes?
6. A speed-time graph rises uniformly from 0 to 20 m/s in 8 s. What is the distance travelled?

Answers to the self-check

1. $\frac{0-9}{5-2} = -3$.
2. Gradient $-\frac{1}{3}$, so $y = -\frac{1}{3}x + 1$.
3. $(x+2)^2 + 3$, so the minimum is 3.
4. $\sqrt{6^2 + 8^2} = 10$.
5. x -axis: $(10, 0)$. y -axis: $(0, -4)$.
6. $\frac{1}{2} \times 8 \times 20 = 80$ m.

Next step

Heurisco's *ESAT Maths 1: Graphs Practice Set* has twenty original exam-style questions on this lesson, in order of difficulty, with a full worked solution for each.