

ESAT

Maths 2

Coordinate geometry

Free lesson notes

Covers

MM3.1 to MM3.3

Format

Lesson, worked examples, self-check

Price

Free

About this lesson

Coordinate geometry turns pictures into algebra. A point becomes a pair of numbers, a line becomes an equation, and a question about shapes becomes a question about solving equations. It is one of the most useful tools in the Maths 2 syllabus, because a good sketch often tells you the answer before any algebra is done.

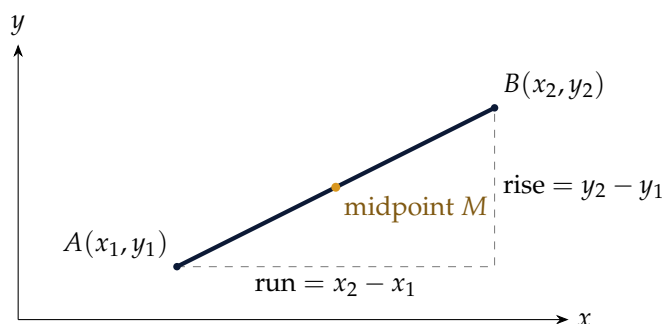
This lesson builds the subject from the ground up. We start with the three basic measurements between two points (gradient, distance and midpoint), then use them to write down equations of straight lines, to handle parallel and perpendicular lines, and to find perpendicular bisectors, intersections and areas. We then derive the equation of a circle from Pythagoras' theorem and study how lines and circles meet: chords, tangents and touching circles. The final section collects problem-solving strategies, including two "intuitive logic" ideas (reflection and maximising along a direction) that ESAT likes to test.

How to use it. Read each idea, then cover the worked example and try it yourself before reading the solution. Always draw a sketch, even a rough one: it reveals whether an answer is sensible. Finish with the self-check, then the practice set. Specification codes: MM3.1 (straight lines), MM3.2 (circles) and MM3.3 (circles and lines).

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1 Measuring between two points

Take two points $A(x_1, y_1)$ and $B(x_2, y_2)$. Moving from A to B we go *across* by $x_2 - x_1$ (the *run*) and *up* by $y_2 - y_1$ (the *rise*). Everything in this section is built from those two numbers.



Gradient. The gradient measures steepness: how far the line rises for each one unit moved to the right.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

A positive gradient slopes up to the right, a negative gradient slopes down, a horizontal line has gradient 0, and a vertical line has no gradient at all (we cannot divide by a run of zero).

Distance. The rise and the run are the two shorter sides of a right-angled triangle whose hypotenuse is AB (the dashed triangle above). Pythagoras' theorem gives

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

It does not matter which point is called A , because squaring removes any minus sign.

Midpoint. The midpoint is half-way across and half-way up, so each coordinate is the average of the two coordinates:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Running the same idea backwards is useful too: if M is the midpoint of AB and you know A and M , then B is as far beyond M as A is before it.

Worked example: gradient, distance and midpoint

For $A(-2, 1)$ and $B(4, 9)$ find the gradient of AB , the length AB , and the midpoint.

Run = $4 - (-2) = 6$ and rise = $9 - 1 = 8$. Gradient = $\frac{8}{6} = \frac{4}{3}$. Length = $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$. Midpoint = $\left(\frac{-2+4}{2}, \frac{1+9}{2}\right) = (1, 5)$.

Worked example: finding an end point from the midpoint

$M(3, -1)$ is the midpoint of AB and $A = (-1, 2)$. Find B .

From A to M we move $+4$ across and -3 up. Moving the same step again from M gives $B = (3 + 4, -1 - 3) = (7, -4)$. (Check: the midpoint of $(-1, 2)$ and $(7, -4)$ is $(3, -1)$.)

Common mistakes

- Subtracting the coordinates in different orders: if you use $y_2 - y_1$ on top, use $x_2 - x_1$ underneath.
- Forgetting to take the square root when finding a distance, or to square the negative terms before adding.
- Adding instead of averaging for the midpoint.

2 Equations of straight lines

A point (x, y) is on a line exactly when its coordinates satisfy the line's equation. There are three standard forms, and the skill is choosing the one that suits the information you are given.

- **Gradient-intercept form** $y = mx + c$. The gradient is m and the line crosses the y -axis at $(0, c)$. Use it when you are told, or can read off, the gradient and the y -intercept.
- **Point-gradient form** $y - y_1 = m(x - x_1)$. This says "from the known point (x_1, y_1) , the rise equals m times the run". It is the most useful form: if you know a point and a gradient, you can write the line at once. To use two points, first find the gradient.
- **General form** $ax + by + c = 0$. Rearranging to $y = -\frac{a}{b}x - \frac{c}{b}$ shows its gradient is $-\frac{a}{b}$ (provided $b \neq 0$). This form is how many answer options are written, and it makes it easy to avoid fractions.

Where does a line cross the axes? It crosses the y -axis where $x = 0$, and the x -axis where $y = 0$. Substitute and solve.

Worked example: a line through two points

Find the equation of the line through $(2, -1)$ and $(6, 7)$, and where it crosses the x -axis.

Gradient = $\frac{7 - (-1)}{6 - 2} = 2$. Point-gradient form with $(2, -1)$: $y + 1 = 2(x - 2)$, so $y = 2x - 5$. It crosses the x -axis when $y = 0$: $2x - 5 = 0$, $x = 2.5$.

Worked example: gradient from general form

Find the gradient and the intercepts of $3x - 2y = 12$.

Rearrange: $-2y = -3x + 12$, so $y = \frac{3}{2}x - 6$. The gradient is $\frac{3}{2}$. Setting $x = 0$ gives $y = -6$; setting $y = 0$ gives $x = 4$. The line passes through $(0, -6)$ and $(4, 0)$.

Worked example: collinear points

The points $(1, 2)$, $(4, k)$ and $(7, 8)$ lie on one straight line. Find k .

Three points are collinear when the gradient between any two of them is the same. From $(1, 2)$ to $(7, 8)$ the gradient is $\frac{6}{6} = 1$. From $(1, 2)$ to $(4, k)$ it is $\frac{k-2}{3}$. Setting these equal gives $k - 2 = 3$, so $k = 5$.

Common mistakes

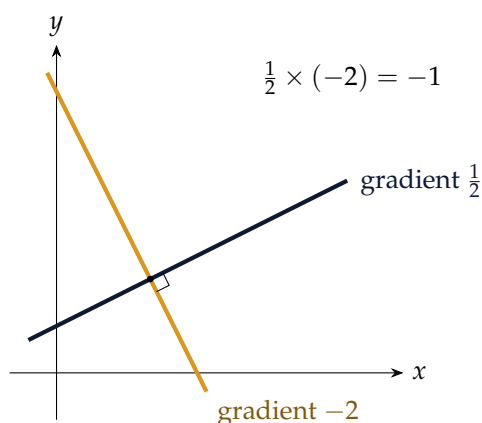
- Reading the number multiplying x as the gradient when the equation has not been rearranged, for example in $3x - 2y = 12$ the gradient is not 3.
- Sign slips in point-gradient form: with the point $(2, -1)$ the left side is $y - (-1) = y + 1$.
- Setting $x = 0$ when the question asks where the line crosses the x -axis (it is $y = 0$).

3 Parallel and perpendicular lines

Two lines are **parallel** when they have the same gradient: $m_1 = m_2$. They are **perpendicular** when

$$m_1 m_2 = -1, \quad \text{i.e. } m_2 = -\frac{1}{m_1}.$$

Why? Draw a right-angled triangle with run 1 and rise m along a line. Turn the page through 90° and the same triangle now has run $-m$ and rise 1, so the new gradient is $\frac{1}{-m} = -\frac{1}{m}$. The perpendicular gradient is therefore the *negative reciprocal*: flip the fraction and change the sign. A horizontal line (gradient 0) is perpendicular to a vertical line, the one case the rule cannot describe.



Distance from a point to a line. The shortest distance from a point to a line is measured along the perpendicular. One method is to find the foot of the perpendicular: write the perpendicular line through the point, solve it simultaneously with the original line, and measure the distance between the point and the foot. For a line $ax + by + c = 0$ and a point (x_0, y_0) there is also a direct formula,

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}},$$

which is worth knowing because it is fast, but the foot-of-perpendicular method always works.

Worked example: a perpendicular through a point

Find the equation of the line through $(2, 1)$ perpendicular to $3x + y = 5$.

Rearranging, $y = -3x + 5$ has gradient -3 , so the perpendicular gradient is $\frac{1}{3}$. Then $y - 1 = \frac{1}{3}(x - 2)$, so $3y - 3 = x - 2$ and $x - 3y = -1$.

Worked example: the foot of a perpendicular

Find the foot of the perpendicular from $P(-1, 2)$ to the line $3x - 2y = 6$, and the distance from P to the line.

The line has gradient $\frac{3}{2}$, so the perpendicular through P has gradient $-\frac{2}{3}$: $y - 2 = -\frac{2}{3}(x + 1)$, i.e. $2x + 3y = 4$. Solve $3x - 2y = 6$ and $2x + 3y = 4$: multiply the first by 3 and the second by 2 and add, $13x = 26$, so $x = 2$ and $y = 0$. The foot is $F(2, 0)$ and $PF = \sqrt{3^2 + 2^2} = \sqrt{13}$. Check with the formula: $\frac{|3(-1) - 2(2) - 6|}{\sqrt{9 + 4}} = \frac{13}{\sqrt{13}} = \sqrt{13}$.

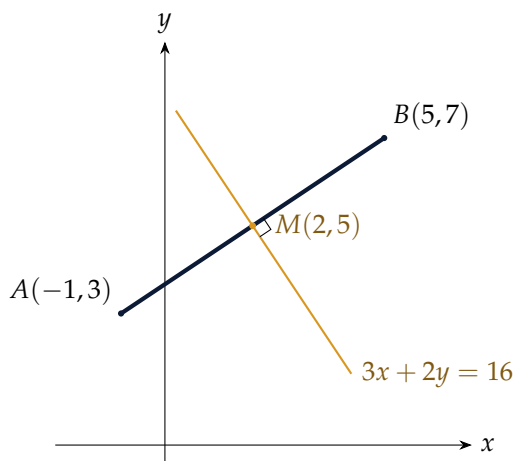
Common mistakes

- Taking the reciprocal but not changing the sign (or the other way round). Check by multiplying the two gradients: the product must be -1 .

- Using the gradient of a line written as $ax + by + c = 0$ as a or $-b$. It is $-a/b$.
- Forgetting the absolute value in the distance formula: a distance is never negative.

4 Perpendicular bisectors, intersections and areas

Perpendicular bisector. The perpendicular bisector of AB cuts AB at right angles through its midpoint. So it needs just two ingredients: the midpoint of AB and the negative reciprocal of the gradient of AB . Its special property is that every point on it is *equidistant* from A and B ; this is why the centre of a circle through A and B must lie on it.



Worked example: perpendicular bisector

Find the perpendicular bisector of $A(-1, 3)$ and $B(5, 7)$.

Midpoint $(2, 5)$. Gradient of $AB = \frac{7-3}{5-(-1)} = \frac{2}{3}$, so the perpendicular gradient is $-\frac{3}{2}$. Then $y - 5 = -\frac{3}{2}(x - 2)$, so $2y - 10 = -3x + 6$ and $3x + 2y = 16$.

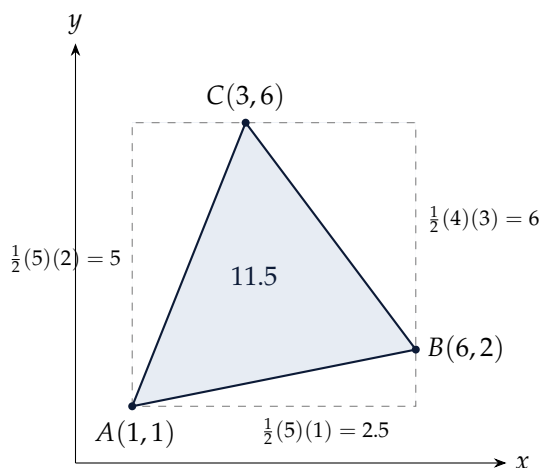
Worked example: the same result from “equidistant”

Show that the points (x, y) equidistant from $A(-1, 3)$ and $B(5, 7)$ satisfy $3x + 2y = 16$.

Equal distances means equal squared distances: $(x + 1)^2 + (y - 3)^2 = (x - 5)^2 + (y - 7)^2$. Expanding, $x^2 + 2x + 1 + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 - 14y + 49$. The squared terms cancel, leaving $12x + 8y = 64$, i.e. $3x + 2y = 16$.

Where two lines meet. A point on both lines satisfies both equations, so solve them as simultaneous equations (substitution or elimination).

Area of a triangle. For a triangle with a horizontal or vertical side, use $\frac{1}{2} \times \text{base} \times \text{height}$. For a general triangle, *enclose it in a rectangle with sides parallel to the axes*, then subtract the three right-angled triangles left in the corners. Alternatively, if $\vec{AB} = (p, q)$ and $\vec{AC} = (s, t)$, the area is $\frac{1}{2} |pt - qs|$.



Worked example: area of a triangle

Find the area of the triangle $A(1,1)$, $B(6,2)$, $C(3,6)$ (shown above).

The enclosing rectangle is $5 \times 5 = 25$. The three corner triangles have areas $\frac{1}{2}(5)(1) = 2.5$, $\frac{1}{2}(4)(3) = 6$ and $\frac{1}{2}(5)(2) = 5$, a total of 13.5. The triangle has area $25 - 13.5 = 11.5$. Check with vectors: $\vec{AB} = (5,1)$, $\vec{AC} = (2,5)$, so the area is $\frac{1}{2}|5 \times 5 - 1 \times 2| = 11.5$.

Worked example: a triangle from two lines and an axis

The lines $y = x + 1$ and $y = -2x + 7$ and the x -axis form a triangle. Find its area.

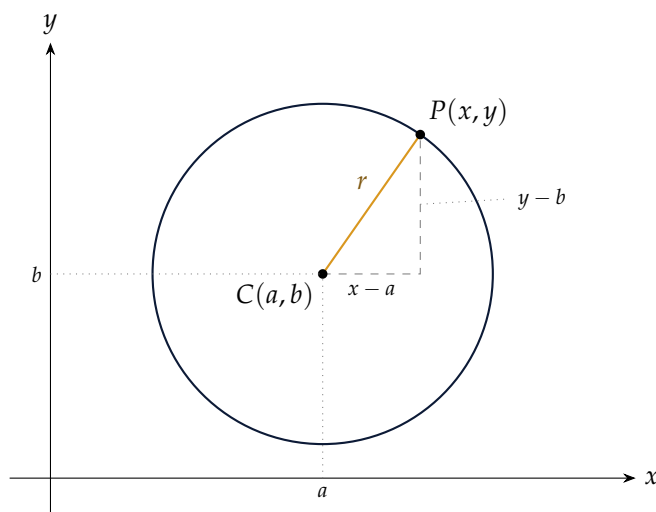
The lines meet where $x + 1 = -2x + 7$, so $x = 2$, $y = 3$. The first line meets the x -axis at $(-1,0)$ and the second at $(3.5,0)$. The base along the axis is $3.5 - (-1) = 4.5$ and the height is the y -coordinate 3 of the meeting point, so the area is $\frac{1}{2}(4.5)(3) = 6.75$.

Common mistakes

- Forgetting the $\frac{1}{2}$ in a triangle's area.
- In the enclosing-rectangle method, forgetting one of the three corner triangles.
- Using a sloping side as the height. The height must be perpendicular to the base.

5 The equation of a circle

A circle is the set of all points at a fixed distance r (the *radius*) from a fixed point $C(a,b)$ (the *centre*). Take a general point $P(x,y)$ on the circle. The horizontal and vertical differences between C and P are $x - a$ and $y - b$, and they form the two shorter sides of a right-angled triangle with hypotenuse $CP = r$.



Pythagoras' theorem therefore gives the equation of the circle:

$$(x - a)^2 + (y - b)^2 = r^2.$$

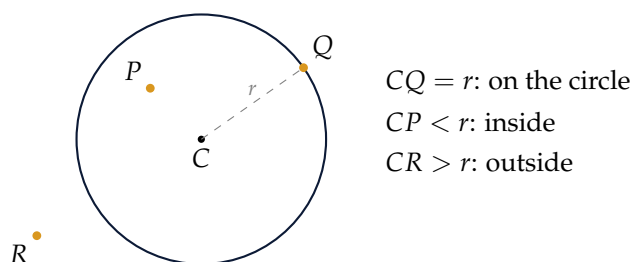
Notice the minus signs: the circle with centre $(3, -2)$ has equation $(x - 3)^2 + (y + 2)^2 = r^2$, so the signs inside the brackets are the *opposite* of the coordinates of the centre. A circle centred at the origin is simply $x^2 + y^2 = r^2$.

The expanded form. Multiplying out gives $x^2 + y^2 + cx + dy + e = 0$, with $c = -2a$, $d = -2b$ and $e = a^2 + b^2 - r^2$. You can recognise a circle in this form because x^2 and y^2 have the *same* coefficient (and there is no xy term). To read off the centre and radius, *complete the square* separately in x and in y . Recall that $x^2 + cx = (x + \frac{c}{2})^2 - \frac{c^2}{4}$, so the centre has coordinates $(-\frac{c}{2}, -\frac{d}{2})$.

Worked example: completing the square

Find the centre and radius of $x^2 + y^2 - 8x + 6y - 11 = 0$.

Group and complete the square: $(x^2 - 8x) + (y^2 + 6y) = 11$ becomes $(x - 4)^2 - 16 + (y + 3)^2 - 9 = 11$, so $(x - 4)^2 + (y + 3)^2 = 36$. The centre is $(4, -3)$ and the radius is 6.



Inside, on or outside. Compare the distance d from a point to the centre with the radius: $d < r$ means inside, $d = r$ on the circle, $d > r$ outside. In practice substitute the point into $(x - a)^2 + (y - b)^2$ and compare the result with r^2 (no square root needed).

Worked example: a circle from a diameter

$A(-1, 0)$ and $B(5, 0)$ are the ends of a diameter. Find the equation of the circle.

The centre is the midpoint of the diameter, $(2, 0)$, and the radius is half the diameter, $\frac{1}{2} \times 6 = 3$. The equation is $(x - 2)^2 + y^2 = 9$.

Worked example: centre on a line

A circle passes through $(1, 2)$ and $(5, 2)$ and its centre lies on $y = x$. Find r^2 .

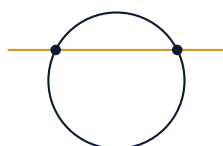
The centre is equidistant from the two points, so it lies on their perpendicular bisector. The chord is horizontal, so its perpendicular bisector is the vertical line $x = 3$. With $y = x$ the centre is $(3, 3)$, and $r^2 = (3 - 1)^2 + (3 - 2)^2 = 5$.

Common mistakes

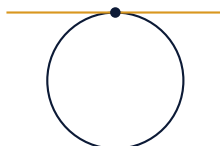
- Writing the centre as $(-a, -b)$ when the equation is $(x - a)^2 + (y - b)^2 = r^2$, or reading the centre of $(x + 2)^2 + \dots$ as $(2, \dots)$. The centre is $(-2, \dots)$.
- Forgetting to square the radius on the right-hand side: $(x - 1)^2 + y^2 = 25$ has radius 5, not 25.
- In completing the square, forgetting to move the squared constants to the right-hand side.

6 Where a line meets a circle

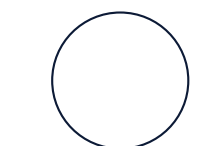
To find where a line meets a circle, write y (or x) from the line's equation, substitute into the circle's equation, and solve the resulting quadratic. The quadratic's discriminant $b^2 - 4ac$ decides the number of intersection points: $\Delta > 0$ gives two points (the line is a *secant*), $\Delta = 0$ gives exactly one (the line is a *tangent*), and $\Delta < 0$ gives none.



two points: $\Delta > 0$



tangent: $\Delta = 0$



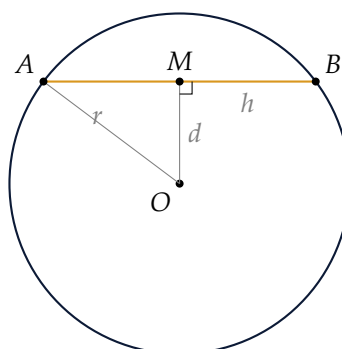
no points: $\Delta < 0$

Do not forget the last step: once you have the x -values, substitute back into the *line* (the easier equation) to find the matching y -values.

Worked example: a chord

The line $x + y = 4$ meets $x^2 + y^2 = 10$. Find the length of the chord.

Substitute $y = 4 - x$: $x^2 + (4 - x)^2 = 10$ gives $2x^2 - 8x + 6 = 0$, i.e. $x^2 - 4x + 3 = 0$, so $x = 1$ or 3 . The points are $(1, 3)$ and $(3, 1)$ and the chord has length $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.



M is the midpoint of AB ,
 $r^2 = d^2 + h^2$

A geometric shortcut for chords. The line from the centre perpendicular to a chord bisects it. If the perpendicular distance from the centre to the line is d , the half-chord h satisfies $r^2 = d^2 + h^2$ (Pythagoras again), so the chord length is $2\sqrt{r^2 - d^2}$. For the chord above: the distance from the origin to $x + y = 4$ is $\frac{4}{\sqrt{2}} = 2\sqrt{2}$, so $h = \sqrt{10 - 8} = \sqrt{2}$ and the chord is $2\sqrt{2}$.

Worked example: when is a line a tangent?

For what positive k is $y = x + k$ a tangent to $x^2 + y^2 = 8$?

Substitute: $x^2 + (x + k)^2 = 8$ gives $2x^2 + 2kx + k^2 - 8 = 0$. A tangent means one repeated root, so $\Delta = (2k)^2 - 4(2)(k^2 - 8) = 64 - 4k^2 = 0$. Then $k = \pm 4$ and $k = 4$ (positive).

Geometric check: the distance from the origin to $x - y + k = 0$ is $\frac{|k|}{\sqrt{2}}$, which equals the radius $2\sqrt{2}$ when $|k| = 4$.

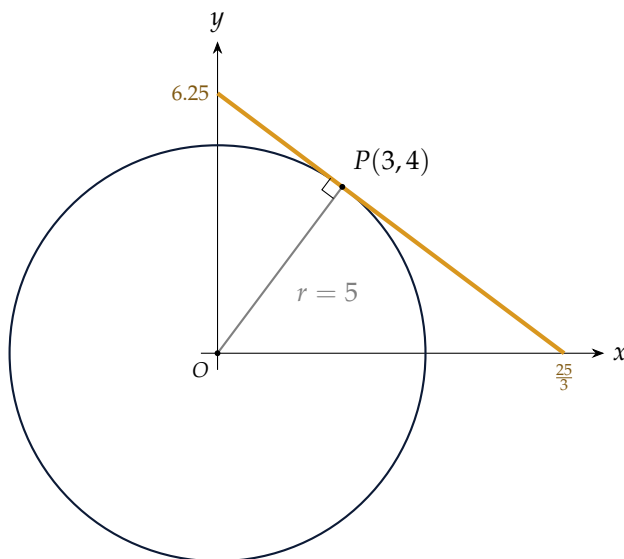
Common mistakes

- Forgetting to substitute back to get the y -coordinates, or substituting into the circle (which gives extra, false pairs).
- Making the sign error $(x + k)^2 = x^2 + k^2$: expand properly as $x^2 + 2kx + k^2$.
- Using the discriminant on a quadratic that has not been fully rearranged to $= 0$.

7 Circle properties: tangents, semicircles and touching circles

Three facts from circle geometry make many problems almost trivial. Learn them as pictures.

1. A tangent is perpendicular to the radius at the point of contact. So the tangent at P has gradient equal to the negative reciprocal of the gradient of CP .

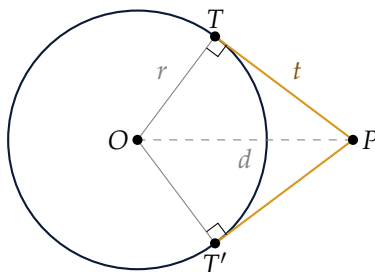


Worked example: a tangent

Find where the tangent to $x^2 + y^2 = 25$ at $(3, 4)$ crosses the y -axis.

The radius to $(3, 4)$ has gradient $\frac{4}{3}$, so the tangent has gradient $-\frac{3}{4}$: $y - 4 = -\frac{3}{4}(x - 3)$. At $x = 0$: $y = 4 + \frac{9}{4} = \frac{25}{4} = 6.25$. (Setting $y = 0$ gives the x -intercept $\frac{25}{3}$. The tangent can be written neatly as $3x + 4y = 25$: for the circle $x^2 + y^2 = r^2$ the tangent at (x_1, y_1) is $x_1x + y_1y = r^2$.)

2. Tangent length. Two tangents drawn from an external point P are equal in length. Each forms a right-angled triangle CTP with the radius, with the right angle at T , so if $d = CP$, the tangent length is $t = \sqrt{d^2 - r^2}$.

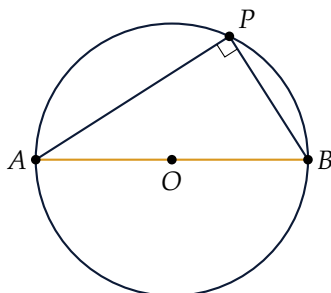


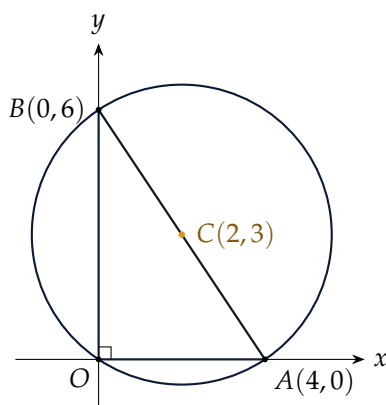
Worked example: tangent length

Find the length of the tangent from $P(10, 0)$ to $x^2 + y^2 = 36$.

$CP = 10$ and $r = 6$, so the tangent length is $\sqrt{10^2 - 6^2} = \sqrt{64} = 8$ (a 6–8–10 right-angled triangle).

3. The angle in a semicircle is a right angle. If AB is a diameter and P is any other point on the circle, then $\angle APB = 90^\circ$. The converse is just as useful: if a triangle has a right angle, the hypotenuse is a diameter of the circle through its vertices, so the centre is the midpoint of the hypotenuse.



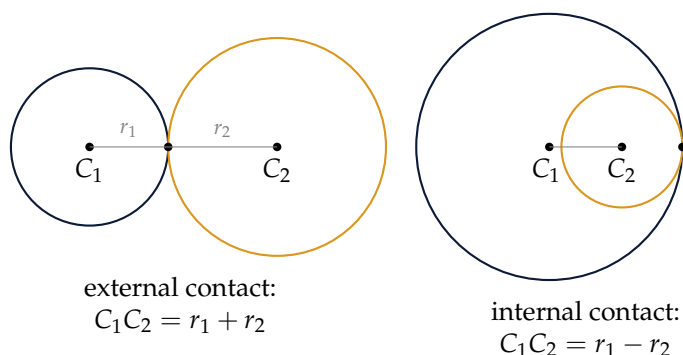


Worked example: a circle through three points

A circle passes through $(0,0)$, $(4,0)$ and $(0,6)$. Find the centre and r^2 .

The angle at the origin is a right angle (the axes are perpendicular), so $(4,0)$ and $(0,6)$ are the ends of a diameter. The centre is their midpoint $(2,3)$, and $r^2 = 2^2 + 3^2 = 13$. Check the other point: $(2-4)^2 + 3^2 = 13$. For three points with no right angle, use the perpendicular bisectors of two of the chords: the centre is where they meet.

4. Touching circles. If two circles touch, the point of contact lies on the line through the centres. For *external* contact the distance between the centres is $r_1 + r_2$; for *internal* contact (one inside the other) it is $|r_1 - r_2|$. If the distance is greater than $r_1 + r_2$ they do not meet; if it is between $|r_1 - r_2|$ and $r_1 + r_2$ they cross at two points.



Worked example: touching circles

Circle C_1 has centre $(0,0)$ and radius 3. Circle C_2 has centre $(5,12)$ and touches C_1 externally. Find its radius.

The distance between the centres is $\sqrt{5^2 + 12^2} = 13$. External contact means $13 = 3 + r$, so $r = 10$. (For internal contact it would be $13 = r - 3$, giving $r = 16$.)

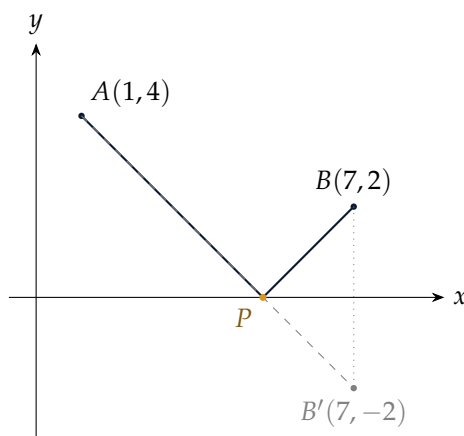
Common mistakes

- Using the gradient of the radius as the gradient of the tangent. They are *perpendicular*.
- Using $CP - r$ or $CP + r$ instead of $\sqrt{CP^2 - r^2}$ for the tangent length.
- Confusing external and internal contact: sum of radii for external, difference for internal.

8 Strategy and two intuitive tricks

A routine for any question. (1) Sketch the situation and mark everything you are given. (2) Decide whether a geometric fact (right angle in a semicircle, tangent perpendicular to radius, 3–4–5 and other Pythagorean triples) answers it quickly. (3) If not, set up equations: a line gives a linear equation, a circle gives a quadratic, and tangency means a repeated root. (4) Check the answer against the sketch: is it plausible?

Trick 1: reflection and shortest routes. Suppose you must walk from A to a line, touch it, and then go on to B , with A and B on the same side of the line. The shortest route is found by *reflecting* B in the line to give B' : a path via the line to B has the same length as the same path to B' (the reflection of the second part), and the shortest distance from A to B' is a straight line. So the best touching point is where AB' crosses the line. This is exactly how light reflects off a mirror.



Worked example: shortest route via the x -axis

A robot goes from $A(1, 4)$ to a point P on the x -axis and then to $B(7, 2)$. Find the shortest possible total distance. Reflect B in the x -axis to get $B'(7, -2)$. Then $AP + PB = AP + PB' \geq AB'$, with equality when P is on the straight line AB' . So the shortest distance is $AB' = \sqrt{6^2 + 6^2} = 6\sqrt{2}$. The line AB' crosses the axis at $P(5, 0)$. Check: $AP + PB = \sqrt{16 + 16} + \sqrt{4 + 4} = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$.

Trick 2: the largest value along a direction. To find the largest possible value of an expression such as $3x + 4y$ for points on a circle, think of the family of parallel lines $3x + 4y = c$. As c increases the line slides across the page, and the largest c that still touches the circle is the one for which the line is a tangent. Start at the centre and move a distance r in the direction perpendicular to the lines: the expression changes by $r\sqrt{3^2 + 4^2} = 5r$ in total.

Worked example: the largest value on a circle

$P(x, y)$ lies on $(x - 4)^2 + (y - 3)^2 = 4$. Find the greatest value of $3x + 4y$. At the centre $(4, 3)$ the expression equals $12 + 12 = 24$. Moving from the centre a distance 2 (the radius) in the direction $(3, 4)/5$, which increases $3x + 4y$ fastest, adds $2 \times 5 = 10$. So the maximum is 34 (and the minimum is 14).

Exam tip

If the numbers in a question make a 3–4–5 or 5–12–13 triangle, a geometric solution with Pythagoras is usually much quicker than substituting and solving a quadratic.

Check yourself

Try these without looking back, then compare with the answers on the next page.

- Find the midpoint and length of the segment joining $(-3, 2)$ and $(5, 8)$.
- Find the equation of the line through $(2, 1)$ perpendicular to $3x + y = 5$.
- Find the centre and radius of $x^2 + y^2 + 6x - 4y - 12 = 0$.
- Is $(2, 6)$ inside, on or outside the circle $(x - 1)^2 + (y - 2)^2 = 16$?
- A line is tangent to $x^2 + y^2 = r^2$ at $(0, 5)$. Give the tangent's equation and r .
- Find the length of the tangent from $(13, 0)$ to $x^2 + y^2 = 25$.
- The ends of a diameter are $(-1, 0)$ and $(5, 0)$. Find the equation of the circle.
- A beam of light goes from $A(0, 3)$ to the x -axis and reflects to $B(6, 1)$. Where does it meet the axis?

Answers to the self-check

1. Midpoint $(1, 5)$. Length $\sqrt{8^2 + 6^2} = 10$.
2. Gradient of given line -3 , so perpendicular gradient $\frac{1}{3}$: $y - 1 = \frac{1}{3}(x - 2)$, i.e. $x - 3y = -1$.
3. $(x + 3)^2 + (y - 2)^2 = 25$. Centre $(-3, 2)$, radius 5.
4. Squared distance $1 + 16 = 17 > 16$: outside.
5. $y = 5$ and $r = 5$ (radius is vertical, so the tangent is horizontal).
6. $\sqrt{169 - 25} = 12$.
7. Centre $(2, 0)$, $r = 3$: $(x - 2)^2 + y^2 = 9$.
8. Reflect B to $B'(6, -1)$. Line AB' has gradient $-\frac{2}{3}$ and crosses $y = 0$ at $x = 4.5$.

Next step

Heurisco's *ESAT Maths 2: Coordinate Geometry Practice Set* has twenty original exam-style questions on this lesson, with diagrams, in order of difficulty, with a full worked solution for each.