

ESAT

Physics

Kinematics

Free lesson notes

Covers	P3.1
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

Kinematics is the science of *how things move*, described using distance, time, speed, velocity and acceleration, without yet asking *why* they move (that is the job of forces, in the next lesson). It is the language every other part of mechanics is written in, and it is a rich source of ESAT questions because it combines three skills: careful definitions, reading and drawing graphs, and quick arithmetic with friendly numbers.

This lesson builds the subject from nothing. We start by separating *distance* from *displacement* and *speed* from *velocity*. Then we meet the three standard graphs (displacement–time, velocity–time and acceleration–time) and learn what their gradients and areas mean. From the velocity–time graph we *derive* the equations of uniform acceleration, so that you never have to memorise them blindly. We apply them to free fall, then to the everyday reasoning that ESAT likes to test: trains passing, overtaking, stopping distances and estimation.

Conventions used. There is no calculator in the ESAT, so all numbers here are friendly. The acceleration of free fall is taken as $g = 10 \text{ m/s}^2$ (the specification says “about 10”). Air resistance is ignored unless the question says otherwise. Specification code covered: P3.1 (scalar and vector quantities; distance–time, displacement–time and velocity–time graphs; average speed; $v^2 - u^2 = 2as$). The other equations of uniform acceleration are *derived* here, because they follow directly from the graph and make $v^2 - u^2 = 2as$ much easier to understand.

How to use it. Read each idea, cover the worked example and try it first, then check. Always sketch a graph or a labelled diagram: in kinematics a ten-second sketch removes most mistakes.

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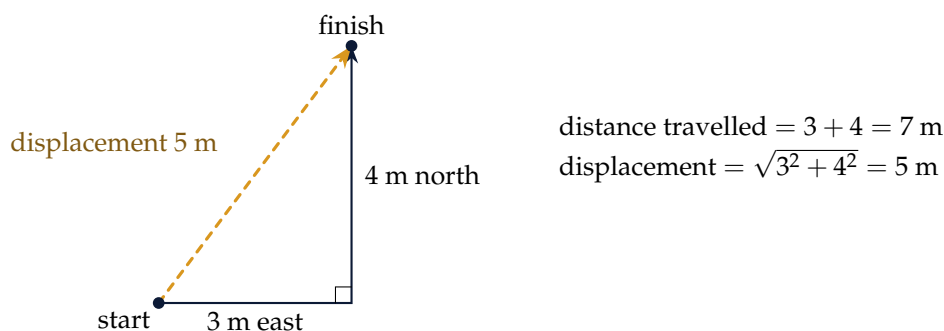
1 Describing motion: distance, displacement, speed, velocity

To describe where something is, we choose a starting point (the *origin*) and a direction. Two different questions can then be asked about a journey.

Distance is *how far you actually travelled along the path*, in total. It counts every metre walked, whichever way you were facing, and it can only grow.

Displacement is *how far you are from where you started, in a straight line, together with the direction*. It cares only about the start and the finish, not the route.

The difference is important enough to have a name. A quantity with only a size (a number and a unit) is a **scalar**: distance, speed, mass, time and energy are scalars. A quantity that also has a *direction* is a **vector**: displacement, velocity, acceleration and force are vectors. In one dimension we show direction with a sign: pick one direction as positive (say *east*, or *up*), and then the opposite direction is negative.



In the diagram a walker goes 3 m east then 4 m north. The *distance* is $3 + 4 = 7 \text{ m}$ because the path is 7 metres long. The *displacement* is the straight line from start to finish. It is the hypotenuse of a right-angled triangle, so by Pythagoras it is

$\sqrt{3^2 + 4^2} = 5$ m, in the direction of the dashed arrow. Distance and displacement are only equal when the motion is in a straight line without turning back.

Speed and velocity. *Speed* measures how quickly distance is covered, and *velocity* measures how quickly displacement changes. For steady motion,

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{velocity} = \frac{\text{displacement}}{\text{time}}.$$

The SI unit of both is metres per second, written m/s (so many metres *every* second). Speed is the size of the velocity. A car going round a roundabout at a steady 10 m/s has constant speed but *changing* velocity, because its direction keeps changing.

Useful conversions. To change km/h into m/s: 1 km = 1000 m and 1 h = 3600 s, so $1 \text{ km/h} = \frac{1000}{3600} = \frac{1}{3.6} \text{ m/s}$. Divide by 3.6 to go from km/h to m/s, and multiply by 3.6 to go back. Two numbers worth remembering: $36 \text{ km/h} = 10 \text{ m/s}$ and $72 \text{ km/h} = 20 \text{ m/s}$.

Worked example: distance and displacement along a line

A jogger runs 6 m east and then turns round and runs 8 m west. What are the distance travelled and the final displacement? Take east as positive.

Distance = $6 + 8 = 14$ m (always add the lengths). Displacement = $(+6) + (-8) = -2$ m: the jogger finishes 2 m *west* of the start.

Worked example: converting units

A cyclist travels at 18 km/h. How far does she go in 20 s?

$18 \div 3.6 = 5 \text{ m/s}$, so in 20 s she travels $5 \times 20 = 100$ m.

Common mistakes

- Treating displacement as if it were distance. A round trip has displacement 0 but a large distance.
- Forgetting that velocity needs a direction: -5 m/s is a speed of 5 m/s in the negative direction.
- Converting km/h to m/s by multiplying by 3.6. It is *divide*. (Sanity check: a metre is much smaller than a kilometre, but a second is much smaller than an hour; the second effect wins and the number gets smaller.)

2 Average speed and average velocity

Real journeys are rarely steady. **Average speed** is the single steady speed that would cover the same distance in the same total time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}.$$

Notice it is *total* distance over *total* time. It is *not* the mean of the speeds you quote at different stages, unless the stages last for equal *times*. The reason is simple: you spend longer at the slow speed, so the slow speed counts for more. In the same way,

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}},$$

which is zero for any journey that ends where it started.

Worked example: the trap of averaging speeds

A car drives 120 km to a town at 60 km/h and returns along the same road at 40 km/h. What is the average speed for the round trip?

Time out = $120 \div 60 = 2$ h. Time back = $120 \div 40 = 3$ h. Total distance = 240 km, total time = 5 h, so the average speed is $240 \div 5 = 48 \text{ km/h}$, not 50. The car spent more time going slowly. (Average velocity for the round trip is 0, because the displacement is zero.)

Worked example: average speed over stages

A pupil walks 600 m at 1.5 m/s, then runs 600 m at 3 m/s. Find the average speed.

Times: $600 \div 1.5 = 400$ s and $600 \div 3 = 200$ s. Average speed = $\frac{1200}{600} = 2 \text{ m/s}$ (not 2.25).

Common mistakes

- Averaging the speeds: $\frac{1}{2}(60 + 40) = 50$ is wrong when equal *distances* are covered at the two speeds.
- Using the *displacement* for an average speed, or the distance for an average velocity.

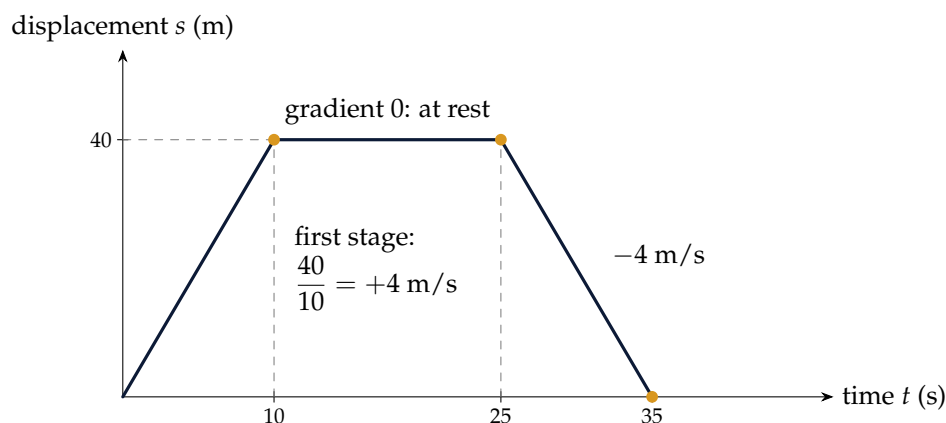
3 Displacement–time (and distance–time) graphs

A graph of position against time is a picture of a whole journey. Time goes along the horizontal axis (it is the quantity we choose to follow), and displacement (or distance) goes up the vertical axis. Because

$$\text{velocity} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{\text{rise}}{\text{run}},$$

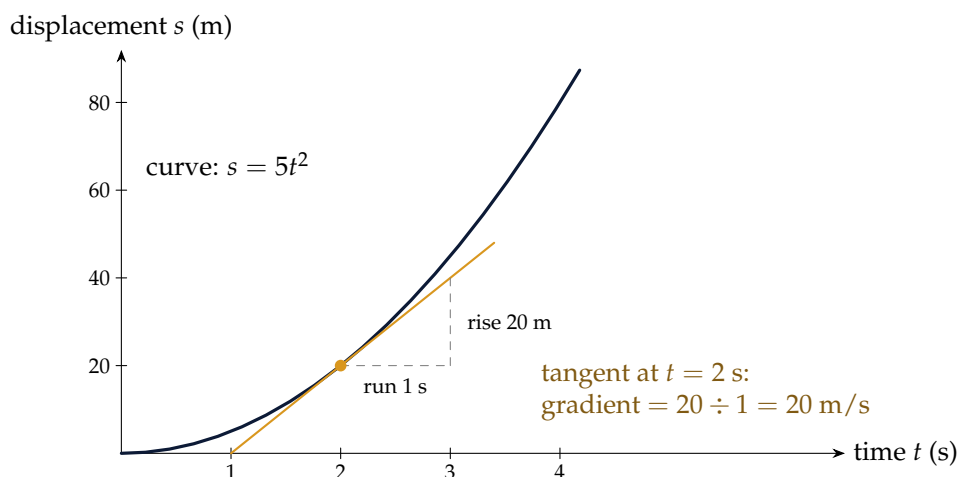
the **gradient of a displacement–time graph is the velocity**. (On a *distance*–time graph the gradient is the speed, and the graph can never go downwards, because distance can only increase.) Reading the shape tells you the story:

- a straight sloping line: constant velocity (the steeper, the faster);
- a horizontal line: gradient zero, so the object is at rest;
- a line sloping down on a displacement graph: negative velocity, moving back towards (or past) the start;
- a curve: the velocity is changing, that is, the object is accelerating. A curve that gets steeper means speeding up; one that flattens means slowing down.



Here the object moves away at $+4 \text{ m/s}$ for 10 s (it travels 40 m), rests for 15 s, then comes back at -4 m/s for 10 s and finishes at the start. Its total *distance* is $40 + 40 = 80 \text{ m}$ in 35 s, so its average speed is $\frac{80}{35} \approx 2.3 \text{ m/s}$, but its average *velocity* is zero.

Curved graphs and tangents. If the graph is a curve, the velocity at one instant (the *instantaneous* velocity) is the gradient of the **tangent**: the straight line that just touches the curve at that point. Choose two clear points on the tangent, far apart, and divide rise by run. Do not use the chord joining the origin to the point: that gives the *average* velocity up to that moment, not the velocity *at* it.



Worked example: velocity from a tangent

The curve above is $s = 5t^2$ (metres, seconds). Find the velocity at $t = 2$ s and compare it with the average velocity over the first two seconds.

The tangent at $t = 2$ passes through $(2, 20)$ and $(3, 40)$, so its gradient is $\frac{40 - 20}{3 - 2} = 20$ m/s. The average velocity over the first two seconds is $\frac{20 - 0}{2 - 0} = 10$ m/s, only half as much: the object was speeding up, so it moves faster at the end of the interval than on average.

Common mistakes

- Reading the *height* of a displacement graph as the speed. The height is where the object is; the *gradient* is how fast it is moving.
- A horizontal line on a displacement–time graph means *at rest*. (On a velocity–time graph it means constant velocity: a very common mix-up, met again below.)
- Using the chord from the origin instead of a tangent to find the speed at an instant.

4 Acceleration and velocity–time graphs

When velocity changes we say the object **accelerates**. **Acceleration** is the rate of change of velocity:

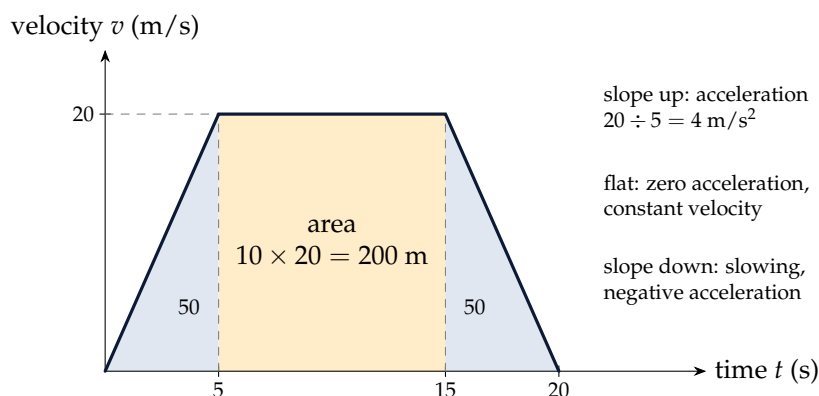
$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t},$$

where u is the starting (initial) velocity, v the final velocity and t the time taken. Its unit is m/s per second, written m/s^2 . An acceleration of 3 m/s^2 means the velocity increases by 3 m/s every second. Acceleration is a vector. If an object moving in the positive direction slows down, its acceleration is negative; this is often called *deceleration*, but in the equations it is simply a negative number.

The velocity–time graph. Plot velocity up and time along. Now two facts follow at once.

(1) **The gradient is the acceleration**, since $\frac{\text{rise}}{\text{run}} = \frac{\Delta v}{\Delta t} = a$.

(2) **The area under the graph is the displacement.** Over a very short time Δt the velocity is almost constant, and displacement $= v \times \Delta t$ is the area of a thin strip of height v and width Δt . Adding all the strips gives the whole area. A rectangle has area (height) $\times$ (width), a triangle $\frac{1}{2}(\text{base})(\text{height})$, a trapezium $\frac{1}{2}(\text{sum of parallel sides})(\text{width})$.



In the figure the object accelerates from rest to 20 m/s in 5 s ($a = 20 \div 5 = 4 \text{ m/s}^2$), cruises for 10 s, then slows to rest in 5 s. The areas are 50, 200 and 50 m, so the total distance is 300 m. The whole shape is a trapezium, and $\frac{1}{2}(10 + 20) \times 20 = 300$ confirms it.

Negative velocity. Parts of the graph *below* the time axis represent motion in the negative direction. Their area counts as *negative* displacement. To find the *displacement*, add areas with their signs; to find the total *distance travelled*, add the sizes of all the areas. If the graph is a curve, its gradient at any point is the acceleration at that instant, and the area can be estimated by counting squares.

Worked example: acceleration and distance from numbers

A train speeds up from 4 m/s to 16 m/s in 6 s. Find its acceleration and the distance covered.

$a = \frac{16 - 4}{6} = 2 \text{ m/s}^2$. The area under its graph is a trapezium: $\frac{1}{2}(4 + 16) \times 6 = 60 \text{ m}$.

Worked example: a graph that goes below the axis

A trolley has velocity +6 m/s for 4 s, then decreases uniformly to -6 m/s over the next 4 s. Find the displacement and the distance after 8 s.

First 24 m forward. In the second stage the velocity falls linearly through zero at $t = 6 \text{ s}$, so it is a positive triangle $\frac{1}{2}(2)(6) = +6 \text{ m}$ and a negative triangle $\frac{1}{2}(2)(-6) = -6 \text{ m}$. Displacement = $24 + 6 - 6 = 24 \text{ m}$ forward. Total distance = $24 + 6 + 6 = 36 \text{ m}$.

Common mistakes

- A horizontal line on a *velocity*-time graph is constant velocity, *not* rest. Rest is a line along the time axis.
- Using the gradient to get distance, or the area to get acceleration. Gradient: a . Area: s .
- Adding all areas together when you want the displacement: areas below the axis are negative.
- Dropping the sign: slowing down in the positive direction means $a < 0$.

5 The equations of uniform acceleration

Many motions have (nearly) constant acceleration: a car braking steadily, a stone falling. For these there are equations linking five quantities: s (displacement), u (initial velocity), v (final velocity), a (acceleration) and t (time). Instead of memorising them, we read them off the velocity-time graph. The graph of constant acceleration is a straight line from height u (at $t = 0$) to height v (at time t).

Equation 1: from the gradient. The gradient is a , and it also equals rise over run = $\frac{v - u}{t}$. So $a = \frac{v - u}{t}$, or

$$v = u + at.$$

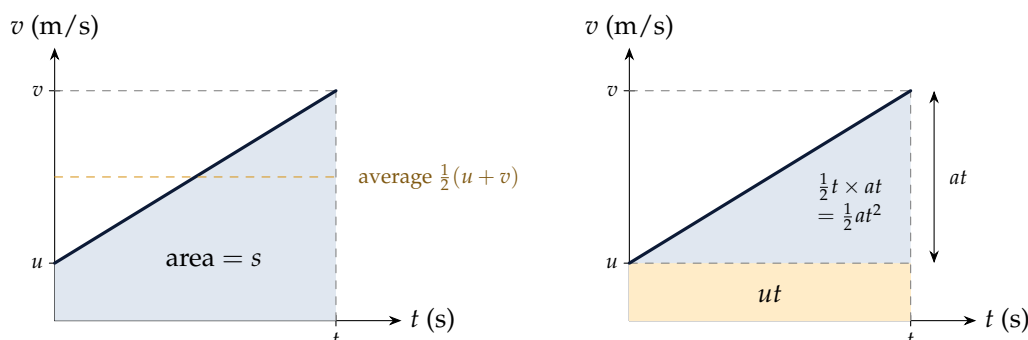
Equation 2: from the area. The shaded shape (left, below) is a trapezium with parallel sides u and v and width t , so

$$s = \frac{1}{2}(u + v)t.$$

Because $\frac{1}{2}(u + v)$ is the average of the starting and finishing velocity, this just says "displacement = average velocity $\times$ time". It is true only when the acceleration is constant.

Equation 3: split the area. Cut the trapezium into a rectangle of height u and width t (area ut) and a triangle on top, of width t and height $v - u = at$ (area $\frac{1}{2} \times t \times at$). Hence

$$s = ut + \frac{1}{2}at^2.$$



Equation 4: remove the time. Sometimes t is neither given nor wanted. Start from equation 2 and substitute $t = \frac{v - u}{a}$ (from equation 1):

$$s = \frac{(u + v)}{2} \times \frac{(v - u)}{a} = \frac{v^2 - u^2}{2a}, \quad \text{so} \quad \boxed{v^2 - u^2 = 2as.}$$

This is the equation named in the specification, and it is the one you will use most.

Which equation? Each equation leaves out one of the five quantities. List what you are given and what you want, find the quantity you have *neither* been given nor asked for, and choose the equation without it:

Equation	Left out	Typical use
$v = u + at$	s	speed after a time
$s = \frac{1}{2}(u + v)t$	a	distance when both speeds are known
$s = ut + \frac{1}{2}at^2$	v	distance after a time
$v^2 - u^2 = 2as$	t	speeds and distances, no time

Signs. Choose a positive direction and stick to it. If an object slows down while moving in the positive direction, then a is negative.

Worked example: braking to rest

A car moving at 20 m/s brakes with constant deceleration 5 m/s² until it stops. Find the braking distance and the time taken.

Take the direction of motion as positive: $u = 20$, $v = 0$, $a = -5$. No time given, so use $v^2 - u^2 = 2as$: $0 - 400 = 2(-5)s$, so $s = 40$ m. Then $v = u + at$ gives $0 = 20 - 5t$, so $t = 4$ s. Check by $s = \frac{1}{2}(u + v)t = \frac{1}{2}(20)(4) = 40$.

Worked example: starting from rest

A cyclist accelerates uniformly from rest at 1.5 m/s² for 8 s. Find her final speed and the distance covered. $u = 0$, $a = 1.5$, $t = 8$. $v = u + at = 0 + 1.5 \times 8 = 12$ m/s. $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(1.5)(64) = 48$ m.

Worked example: finding an acceleration without a time

A ball rolls from 3 m/s to 9 m/s over a distance of 24 m. Find its acceleration. $v^2 - u^2 = 2as$: $81 - 9 = 2a(24)$, so $72 = 48a$ and $a = 1.5$ m/s².

Exam tip

If a question mentions a distance but no time, reach for $v^2 - u^2 = 2as$. A key consequence: starting from the same speed with the same braking, the stopping distance is proportional to u^2 . Doubling the speed makes the stopping distance *four* times as large.

Common mistakes

- Using the equations when the acceleration is *not* constant (for example when a graph has a curved velocity line).
- Writing $v^2 - u^2$ as $(v - u)^2$.
- Forgetting that the final speed of a car which stops is $v = 0$, and that the start speed of something dropped is $u = 0$.
- Mixing units: convert km/h to m/s before using the equations.

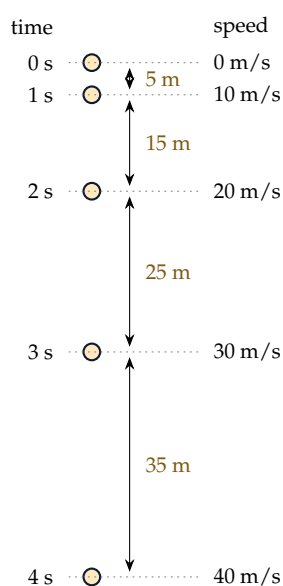
6 Free fall and vertical motion

An object moving under gravity alone (no air resistance) is in **free fall**. Whatever its mass, it has the same constant downward acceleration, the **acceleration of free fall**

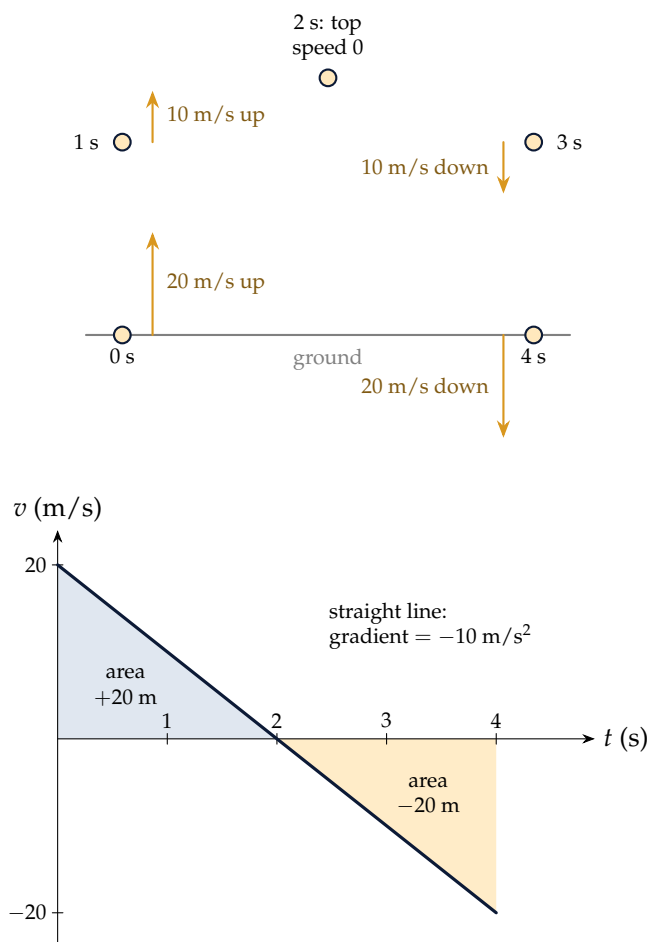
$$g \approx 10 \text{ m/s}^2 \quad (\text{we use } 10; \text{ the true value is about } 9.8).$$

(Why do heavy and light objects fall together? A heavier object is pulled harder, but it also has more mass to move, and the two effects cancel exactly. We prove this in the lesson on forces.) Free fall is just uniform acceleration with $a = g$, so the equations of the previous section apply.

Dropped from rest. Taking down as positive, $u = 0$ and $a = 10$, so $v = 10t$ and $s = \frac{1}{2}(10)t^2 = 5t^2$. After 1, 2, 3, 4 seconds the speed is 10, 20, 30, 40 m/s and the distance fallen is 5, 20, 45, 80 m. The distance covered in each *successive* second grows as 5, 15, 25, 35 m: the odd numbers, because the object is moving faster all the time.



Thrown upwards. Now take *up* as positive: $a = -10$. The velocity decreases by 10 m/s every second, reaches zero at the top, then becomes negative as the object falls back. The velocity–time graph is a single straight line with gradient -10 all the way, even at the top (where the velocity is momentarily zero, the acceleration is still -10). Symmetry gives several shortcuts: the time up equals the time down, and the object returns to its starting height with the same speed that it was thrown with (in the opposite direction). In the example below, the area above the axis (+20 m, up) cancels the area below (–20 m, down) at $t = 4$ s: the ball is back at the start.



Key results for a throw upwards at speed u (use $v = 0$ at the top): time to the top $t = \frac{u}{g}$; greatest height $h = \frac{u^2}{2g}$; total flight time (back at the same level) $\frac{2u}{g}$.

Worked example: a ball thrown straight up

A ball is thrown vertically upwards at 20 m/s. Find the time to the top, the greatest height, and the time it spends above 15 m.

Top: $0 = 20 - 10t$, so $t = 2$ s. Height: $0 - 400 = 2(-10)h$, so $h = 20$ m. For the height 15 m: $15 = 20t - 5t^2$, i.e. $t^2 - 4t + 3 = 0$, so $t = 1$ or 3 s. The ball is above 15 m between 1 s and 3 s: for 2 s. (Check by symmetry: the top is at 2 s, so the times are symmetric about it.)

Worked example: a throw from a cliff

A stone is thrown vertically upwards at 10 m/s from the edge of a cliff 15 m above the beach. When does it land, and how fast is it going then?

Take up as positive. The landing point is 15 m *below* the start, so $s = -15$: $-15 = 10t - 5t^2$, i.e. $t^2 - 2t - 3 = 0$, so $(t - 3)(t + 1) = 0$ and $t = 3$ s (reject $t = -1$). Then $v = 10 - 10 \times 3 = -20$ m/s: a speed of 20 m/s downwards. Check: $v^2 = 100 + 2(-10)(-15) = 400$.

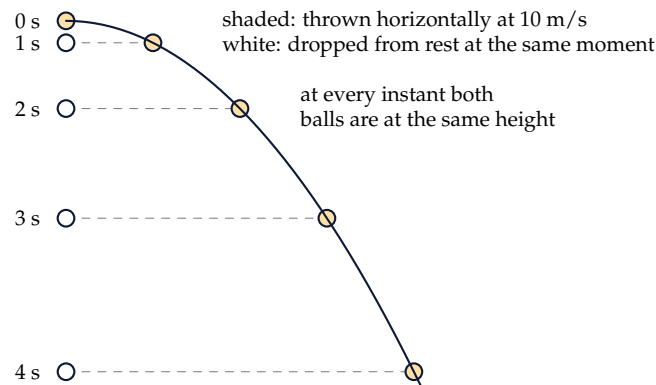
Exam tip

For *any* projectile thrown straight up or down at the same speed from the same height, the landing speed is the same. To see why, use $v^2 = u^2 + 2gh$: the direction of u does not matter, only u^2 .

Beyond the specification (optional)

Thrown sideways. The specification stays in one dimension, but one idea is worth a glance. The vertical motion of a ball thrown horizontally is exactly the same as that of a ball simply dropped, and the horizontal motion goes on

at a constant speed (nothing pushes it sideways). The figure compares the two at one-second intervals.



Common mistakes

- Giving the acceleration at the top of the flight as zero. The velocity is zero there, but the acceleration is still g downwards.
- Mixing up directions: if up is positive, g enters as -10 and a landing below the start has negative displacement.
- Trying to use $s = 5t^2$ for a ball that starts with a throw. That formula is for objects dropped from rest.

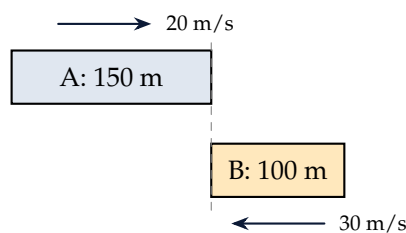
7 Relative motion and everyday reasoning

ESAT likes questions where a little thought replaces a lot of algebra. The most useful idea is **relative speed**: how quickly two moving objects approach or separate from each other.

- Moving *towards* each other (or apart): the gap changes at the **sum** of the speeds.
- Moving in the *same* direction: the gap changes at the **difference** of the speeds.

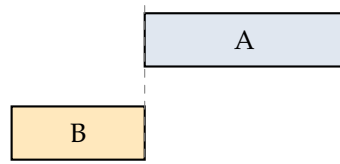
To see why, imagine you are standing on one of them: you see the other approaching at the speed *relative* to you. Now a train problem: how long does a train take to pass a pole, a platform, or another train? What matters is the total length that must slide past, divided by the relative speed.

Start: the fronts are level



A moved 100 m and B moved 150 m, together $250 \text{ m} = 150 + 100$.

End: the rears are level, 5 s later



Worked example: two trains passing

Train A is 150 m long and travels east at 20 m/s. Train B is 100 m long and travels west on the neighbouring track at 30 m/s. How long do they take to pass completely, from fronts level to rears level?

Relative speed = $20 + 30 = 50$ m/s. Relative distance to cover = $150 + 100 = 250$ m (each train must clear the other's whole length). Time = $250 \div 50 = 5$ s.

Worked example: overtaking

A car 4 m long overtakes a lorry 16 m long. The car's speed is 25 m/s and the lorry's is 20 m/s. How long does the overtaking take (front of car level with the lorry's rear, to car's rear level with the lorry's front)?

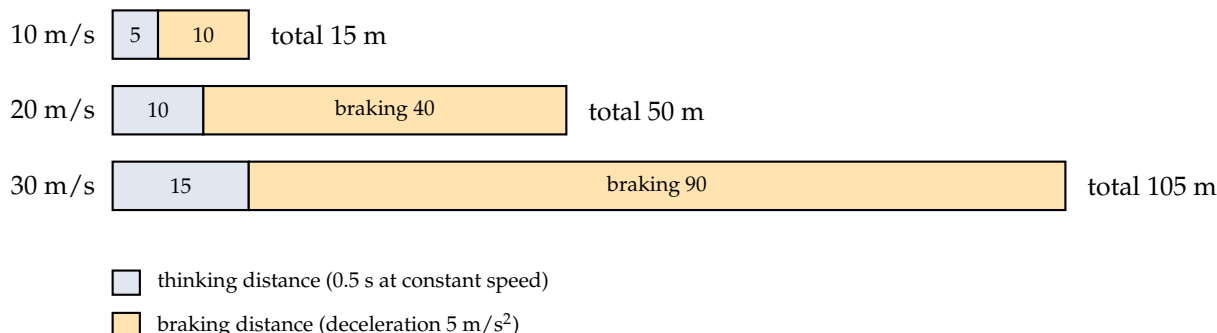
Relative speed = $25 - 20 = 5$ m/s. The car must gain $4 + 16 = 20$ m, so $t = 20 \div 5 = 4$ s.

Worked example: a river journey: the trap of "it cancels"

A boat has a speed of 4 m/s in still water. It goes 120 m downstream and back, in a river flowing at 1 m/s. Find the time for the round trip and compare it with still water.

Downstream the speeds add: 5 m/s, so $120 \div 5 = 24$ s. Upstream the current opposes: 3 m/s, so $120 \div 3 = 40$ s. Total = 64 s. In still water: $240 \div 4 = 60$ s. The current always makes the round trip *longer*, because more time is spent going slowly (the same effect as in the averaging example in section 2).

Stopping distances. The distance a vehicle needs to stop has two parts: the *thinking distance* (how far it travels during the driver's reaction time, at the original speed: distance = speed $\times$ reaction time) and the *braking distance* (found from $v^2 = u^2 + 2as$ with $v = 0$: braking distance = $\frac{u^2}{2a_{\text{dec}}}$, where a_{dec} is the size of the deceleration). Thinking distance is proportional to speed. Braking distance is proportional to speed *squared*. This is why speed limits matter so much: doubling the speed quadruples the braking distance.



Worked example: estimating a reaction time

A ruler is dropped and caught by a student after it has fallen 20 cm. Estimate the reaction time.

Dropped from rest, $s = 5t^2$. So $0.20 = 5t^2$, $t^2 = 0.04$ and $t = 0.2$ s. (A typical human reaction time is about 0.2 to 0.3 s.)

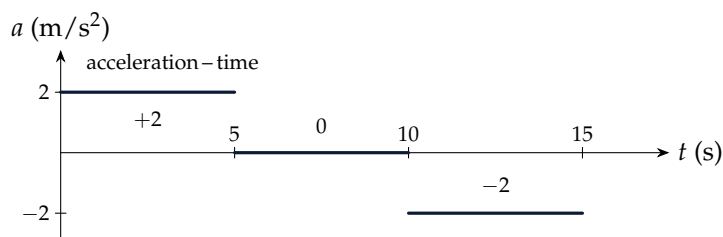
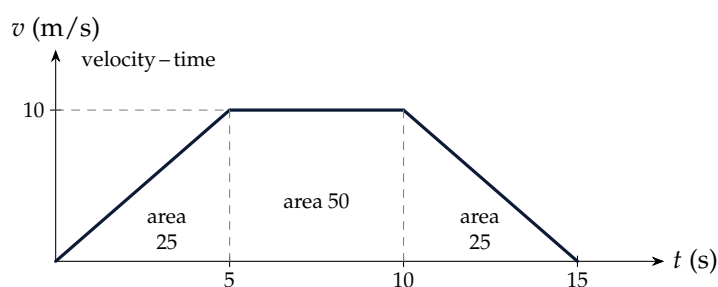
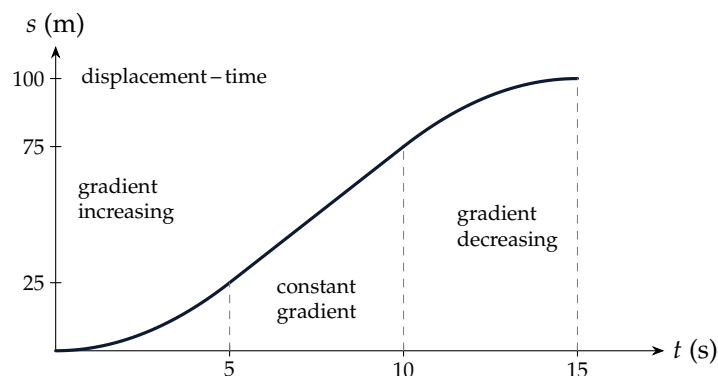
Typical speeds worth knowing (for estimation): walking about 1.5 m/s; running about 4 to 5 m/s; cycling about 5 m/s; a car in town about 13 m/s (30 mph); on the motorway about 30 m/s (70 mph); a passenger jet about 250 m/s; sound in air about 340 m/s.

Common mistakes

- Subtracting speeds for objects moving towards each other. Towards: *add*. Same direction: *subtract*.
- Counting only one train's length when two long trains pass.
- Doubling the stopping distance when the speed doubles. Only the thinking part doubles; the braking part quadruples.

8 Putting the graphs together, and strategy

The three graphs describe the same journey in three different ways, and each is the *gradient* of the previous one and the *area under* the next. Here a cyclist accelerates for 5 s at +2, cruises at 10 m/s for 5 s, then slows to rest at -2:



Graph	Gradient gives	Area gives
displacement-time	velocity	(nothing useful)
velocity-time	acceleration	displacement
acceleration-time	(nothing useful)	change in velocity

Check the displacement graph against the velocity graph: its gradient is 0 at the start (speed zero), 10 m/s in the middle, and back to 0 at the end, and the total rise is $25 + 50 + 25 = 100$ m, exactly the area under the velocity graph.

A routine for kinematics problems.

1. Draw a quick sketch: a velocity-time graph is usually best, with every given number labelled.
2. Choose and write down a positive direction. Convert all units to metres and seconds.
3. Decide whether the acceleration is constant. If so, list s, u, v, a, t : three known, one wanted, one unneeded. Choose the equation that omits the unneeded one.
4. If the motion has several stages, treat each stage separately: the final velocity of one stage is the initial velocity of the next.
5. Check that your answer is sensible (sign, size, units).

Worked example: an unknown speed from the total distance

A tram travels 1.2 km between stops in 100 s. It accelerates uniformly from rest for 20 s, travels at a steady speed v , then brakes uniformly to rest in 20 s. Find v .

The graph is a trapezium: the steady stage lasts $100 - 20 - 20 = 60$ s. Total area = $\frac{1}{2}v(20) + 60v + \frac{1}{2}v(20) = 80v$. So $80v = 1200$ and $v = 15$ m/s. (Equivalently, area of trapezium = $\frac{1}{2}(100 + 60)v = 80v$.)

Worked example: a chase solved with areas

Car A passes a junction at a steady 10 m/s. At that instant car B starts from rest at the same junction with acceleration 2 m/s^2 . When does B catch A, and how fast is B then?

They have travelled equally far when the areas under their velocity graphs are equal. A: $10t$; B: $\frac{1}{2}(2)t^2 = t^2$. So $t^2 = 10t$ and $t = 10$ s (the other solution $t = 0$ is the start). B's speed is $2 \times 10 = 20$ m/s: exactly twice A's, because B's graph is a straight line from the origin and equal areas force it to reach twice the height of A's flat line.

Exam tip

A chase problem means *equal distances*, which means *equal areas* under the two velocity–time graphs. Sketching the two graphs usually reveals the answer, such as “twice the speed”, faster than any algebra.

Common mistakes

- Reading an acceleration off a displacement–time graph. Acceleration is the gradient of the *velocity* graph.
- Using one equation for a whole journey with several stages of different acceleration.
- Forgetting that “from rest” means $u = 0$ and “comes to rest” means $v = 0$.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. A runner completes one lap of a 400 m track in 80 s. State her average speed and average velocity.
2. Convert 54 km/h to m/s.
3. A displacement–time graph is a horizontal line. What can you say about the object? What if the line were on a velocity–time graph?
4. A car's velocity–time graph is a straight line from 0 m/s at 0 s to 12 m/s at 4 s. Find the acceleration and the distance travelled.
5. A car moving at 30 m/s brakes at 6 m/s^2 to rest. How far does it travel while braking?
6. A stone is dropped from rest from the top of a well and takes 3 s to hit the water. How deep is the well, and how fast is the stone going?
7. Two cyclists 300 m apart ride directly towards each other at 4 m/s and 6 m/s. When do they meet?
8. A car travelling at 15 m/s has stopping distance 30 m made up of 9 m thinking distance and 21 m braking distance. Estimate its stopping distance at 30 m/s with the same reaction time and deceleration.

Answers to the self-check

1. Average speed = $400 \div 80 = 5 \text{ m/s}$. The displacement over a full lap is zero, so her average velocity is 0.
2. $54 \div 3.6 = 15 \text{ m/s}$.
3. Displacement graph: the object is at rest (gradient 0). Velocity graph: the object moves at constant velocity (acceleration zero).
4. $a = 12 \div 4 = 3 \text{ m/s}^2$. Distance = $\frac{1}{2} \times 4 \times 12 = 24 \text{ m}$.
5. $0 - 900 = 2(-6)s$, so $s = 75 \text{ m}$.
6. $s = 5 \times 9 = 45 \text{ m}$; $v = 10 \times 3 = 30 \text{ m/s}$.
7. Relative speed 10 m/s , so $t = 300 \div 10 = 30 \text{ s}$.
8. Thinking distance doubles to 18 m ; braking distance quadruples to 84 m . Total = 102 m (not 60).

Next step

Heurisco's *ESAT Physics: Kinematics Practice Set* has twenty original exam-style questions on this lesson, with diagrams, ordered from gentle to genuinely hard, each with a full worked solution and a note on the commonest trap.