

ESAT

Physics

Momentum, energy and power

Free lesson notes

Covers	P3.6 and P3.7
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

Two ideas run through almost all of mechanics, and the ESAT tests both relentlessly: **energy** (how much “doing” a system is capable of, and how it moves between stores) and **momentum** (how much “motion” a system has, and how it is shared out when things collide or push apart). Both are *conserved*: they cannot be created or destroyed, only moved around. That single fact lets you solve problems that would be miserable with forces and accelerations alone, such as the speed of a roller-coaster at the bottom of a dip or the recoil of a gun, without knowing anything about what happens in between.

We build everything from first principles. We start with *work*, the way energy is transferred by a force, and use it to derive the formulas for gravitational potential energy and kinetic energy rather than just quoting them. Then comes conservation of energy, power and efficiency. The second half defines momentum, shows why it is conserved, applies that to collisions and explosions, and finishes with the link between force and change of momentum, which explains airbags, crumple zones and bending your knees when you land.

How to use it. Cover each worked example and try it before reading the solution. There is no calculator in the real ESAT, so the numbers here are friendly: practise doing the arithmetic by hand. We use $g = 10 \text{ N/kg}$ throughout, as the specification does. Finish with the self-check, then the practice set.

Heurisco is an independent practice provider and is not affiliated with, endorsed by, or connected to UAT-UK, Pearson VUE, the UCAT Consortium, or any university. All questions are original.

1 Work: how a force transfers energy (P3.7)

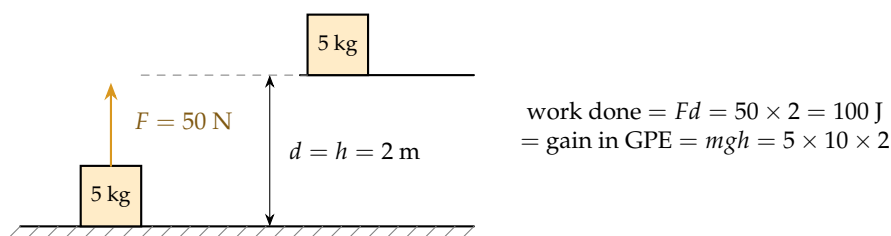
Energy is the quantity that has to be supplied to make things happen: to lift a load, to speed something up, to warm something. It is measured in **joules** (J). Energy is stored in different ways (kinetic, gravitational potential, elastic, thermal, chemical) and moves from one store to another. We cannot see energy itself, so we track it by what changes.

One of the main ways energy is moved is **work**. When a force moves something through a distance *in the direction of the force*, the force does work on it, and that amount of energy is transferred to the object:

$$W = Fd \quad (\text{work done} = \text{force} \times \text{distance moved in the direction of the force}).$$

W is in joules, F in newtons and d in metres. So **one joule is one newton-metre**: the work done by a force of 1 N moving something 1 m.

Why this makes sense. Push a heavy crate along the floor with a steady force. Push it twice as far and you have to keep pushing for twice as long, so you supply twice the energy: $W \propto d$. Push twice as hard over the same distance and you again supply twice the energy: $W \propto F$. Putting these together gives $W = Fd$.



In the picture a 5 kg block is lifted steadily (so the lifting force equals the weight, 50 N) through 2 m. The work done is $50 \times 2 = 100 \text{ J}$, and that energy ends up stored in the block as gravitational potential energy.

Key ideas

- Work done $W = Fd$, measured in joules. The distance must be measured *along the line of the force*.
- Work done on an object is energy transferred to it. If the object transfers energy to something else (such as

friction heating the floor), it is work done *against* that force.

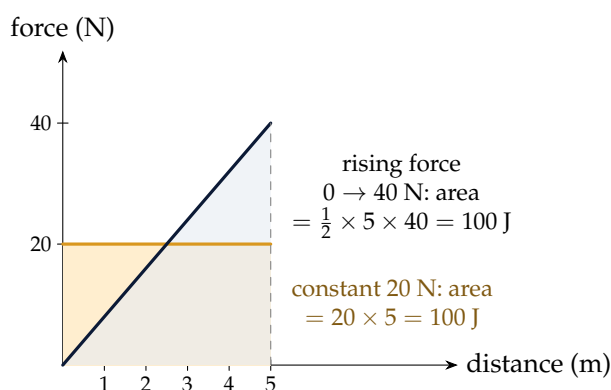
- A force at right angles to the motion does *no* work: it moves no energy.

Worked example: dragging a crate

A worker drags a crate 12 m across a floor with a steady horizontal force of 150 N. How much work does the worker do, and where does the energy go?

$W = Fd = 150 \times 12 = 1800 \text{ J}$. The crate moves at a steady speed, so its kinetic energy is not changing; the energy is transferred by friction as thermal energy to the crate and the floor.

A varying force. If the force changes along the way, we cannot just multiply. But a graph of force against distance shows the answer: the work done for each tiny step is (force) $\times$ (small distance), which is a thin strip of area under the graph. Adding up all the strips gives the **area under a force–distance graph** = work done.



The two lines in the graph tell very different stories, a constant 20 N and a force that rises steadily from 0 to 40 N, but both give 100 J over 5 m because the average force is 20 N in each case. For a force that rises uniformly from zero (as in a stretching spring) the area is a triangle, $\frac{1}{2} \times \text{base} \times \text{height}$. For a mixed graph, split it into rectangles and triangles.

Common mistakes

- Using the *weight* of the object as the force when it is being pushed along the ground. The force that matters is the push, not the weight.
- Measuring distance in the wrong direction. Carrying a bag *horizontally* at constant speed does no work on the bag by the lifting force, since the upward force is at right angles to the motion.
- Forgetting that “work done” and “energy transferred” are the same quantity, so they share the unit joule.

2 Gravitational potential energy (P3.7)

Lift an object and you do work against gravity, so it gains energy that is *stored* in the gravitational field: **gravitational potential energy** (GPE). Let us work out how much.

To lift an object of mass m at a steady speed we must pull up with a force equal to its weight, $F = mg$ (if the pull were bigger, it would accelerate). Lifting it through a height h then does work $W = Fd = mg \times h$. All of it is stored, so

$$\Delta E_p = m g \Delta h \quad (\text{change in GPE} = \text{mass} \times g \times \text{change in height}).$$

Here $g = 10 \text{ N/kg}$ is the gravitational field strength (the weight of each kilogram), m is in kilograms and h in metres. The result is in joules.

Notice what is missing: the path. The formula depends only on the *vertical* height gained. Lift a box straight up 2 m, or push it up a long gentle ramp that rises 2 m (with no friction), and the GPE gained is the same. The ramp needs a smaller force but a longer distance: force $\times$ distance comes out the same.

Only changes matter. We can measure height from anywhere (the floor, the table, sea level): the quantity that matters is the *change*. Falling reverses the process: an object that falls loses GPE as it goes down, and we shall see in the next sections where that energy goes.

Worked example: climbing the stairs

A student of mass 60 kg climbs a staircase that rises 3 m. What GPE does she gain?
 $\Delta E_p = mgh = 60 \times 10 \times 3 = 1800 \text{ J}$.

Worked example: how high for 500 J?

How high can a 2 kg ball be lifted with 500 J of energy, if it is all stored as GPE?
 Rearrange $E_p = mgh$ to $h = \frac{E_p}{mg} = \frac{500}{2 \times 10} = 25 \text{ m}$.

Common mistakes

- Forgetting to multiply by g and giving mh . A mass in kilograms is not a force: its weight is mg .
- Using the *length* of a slope instead of the *vertical* height.
- Using the height of the ground as zero when the question measures from a table: only the *difference* in height matters.

3 Kinetic energy (P3.7)

A moving object has energy because of its motion, its **kinetic energy** (KE). To find the formula, ask how much work it takes to get an object of mass m from rest up to a speed v .

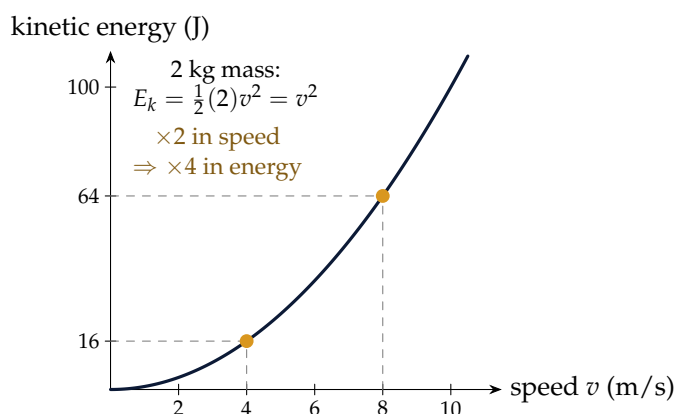
Apply a constant resultant force F over a distance s . By Newton's second law $F = ma$, and for constant acceleration from rest $v^2 = 2as$, so $s = \frac{v^2}{2a}$. The work done is

$$W = Fs = ma \times \frac{v^2}{2a} = \frac{1}{2}mv^2.$$

The a cancels, which is why the answer does not depend on *how* we got up to speed. That work is stored as kinetic energy:

$$E_k = \frac{1}{2}mv^2.$$

Two features matter enormously in questions. First, the energy is proportional to the mass: double the mass, double the KE. Second, it is proportional to the *square* of the speed: **double the speed and the KE quadruples**.



The graph, for a 2 kg mass, shows the curve $E_k = v^2$. At 4 m/s the energy is 16 J; at 8 m/s it is 64 J, four times as much.

Road safety. A moving car must lose all its KE to stop. A braking force F does work Fs over the braking distance s , so $Fs = \frac{1}{2}mv^2$ and $s \propto v^2$. At twice the speed the *braking distance is four times as long* (for the same braking force). This is why the stopping distances in the Highway Code grow so fast with speed.

Worked example: kinetic energy of a cyclist

A cyclist and bike have a total mass of 80 kg and travel at 5 m/s. Find the KE, and then the KE at 10 m/s.
 $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 80 \times 25 = 1000 \text{ J}$. At twice the speed, KE is four times as big: 4000 J. Check: $\frac{1}{2} \times 80 \times 100 = 4000$.

Worked example: braking distance

A car braking with a constant force stops from 15 m/s in 18 m. What distance does it need from 30 m/s with the same force?

Doubling the speed multiplies the KE by 4, so the work the brakes must do is four times as big. The force is the same, so the distance is four times as long: $4 \times 18 = 72$ m.

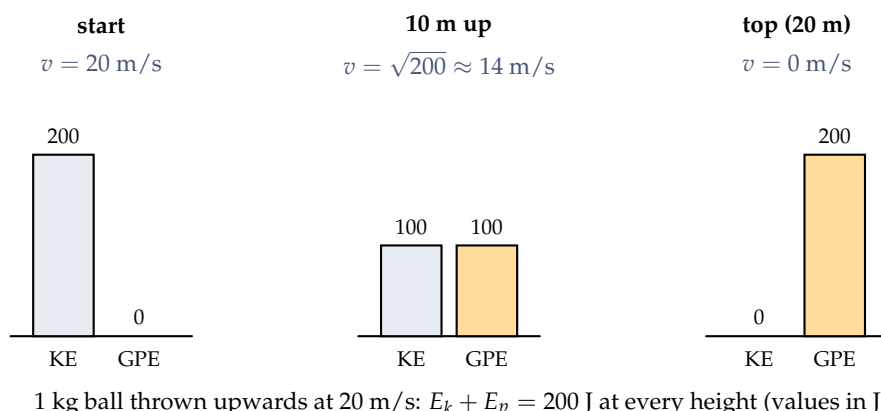
Common mistakes

- Forgetting to *square* the speed, or squaring the $\frac{1}{2}$ along with it. Compute v^2 first, then multiply by $\frac{1}{2}m$.
- Thinking “twice the speed means twice the energy”. It means four times the energy, and four times the braking distance.
- Using the speed in km/h. Convert to m/s (divide by 3.6) first.

4 Conservation of energy (P3.7)

The most powerful statement in physics is that **energy is never created or destroyed**: in any process, the total energy before equals the total energy after. Energy only moves between stores or is transferred between objects. When you think energy has “disappeared” (a ball stops bouncing; a car brakes) it has gone into the surroundings as thermal energy (and a little sound), spread among so many particles that we cannot get it back.

Energy conversion in a thrown ball. A 1 kg ball is thrown straight up at 20 m/s. Ignoring air resistance, the only force doing work is gravity, so KE and GPE trade places and their *sum* is constant.

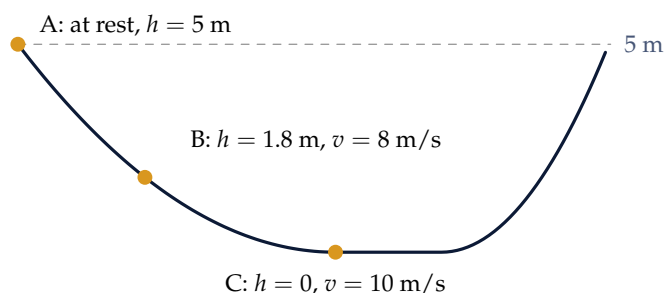


At the start $E_k = \frac{1}{2} \times 1 \times 20^2 = 200$ J and $E_p = 0$. At the top the ball stops (all KE is gone), so $mgh = 200$, giving $h = 20$ m. Half-way up the ball has gained 100 J of GPE and so has lost 100 J of KE, with a speed of $\sqrt{200} \approx 14$ m/s. The bar chart shows the energy being shuffled from one store to the other while the total stays at 200 J.

The standard method. For a mechanical problem with no friction or other losses:

1. Pick two positions (start and finish) and list the energy stores at each.
2. Write: total at start = total at finish.
3. Solve. The mass often cancels, because both mgh and $\frac{1}{2}mv^2$ contain m .

Cancelling gives, for something dropped from rest through a height h with no losses, $\frac{1}{2}mv^2 = mgh$, so $v = \sqrt{2gh}$.



In the track shown, a ball is released from rest at A, 5 m above the lowest level. At C (the lowest point) all the GPE has become KE: $v = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$. At B, only 1.8 m above the bottom, the ball has fallen through $5 - 1.8 = 3.2 \text{ m}$, so $v^2 = 2 \times 10 \times 3.2 = 64$ and $v = 8 \text{ m/s}$. Notice that the *shape* of the track is irrelevant: only the heights matter. Also, the mass cancels, so a heavier ball would reach the same speed.

Worked example: speed after a fall

A stone is dropped from a bridge 20 m above a river. Ignoring air resistance, with what speed does it hit the water? GPE lost = KE gained: $mgh = \frac{1}{2}mv^2$, so $v^2 = 2gh = 2 \times 10 \times 20 = 400$ and $v = 20 \text{ m/s}$.

Worked example: friction on a slope

A 3 kg block slides from rest down a rough slope that falls 2 m vertically, reaching the bottom at 4 m/s. How much energy is lost as thermal energy?

GPE lost = $3 \times 10 \times 2 = 60 \text{ J}$. KE gained = $\frac{1}{2} \times 3 \times 16 = 24 \text{ J}$. The difference, $60 - 24 = 36 \text{ J}$, is the work done against friction, shared as thermal energy between the block and slope.

Exam tip

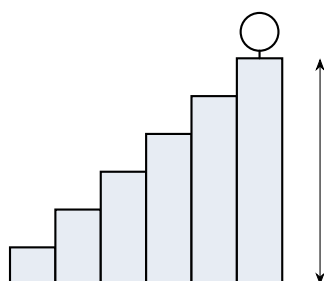
If a question says "smooth", "frictionless" or "ignore air resistance", conservation of energy is the intended route. If it gives a final speed that is smaller than $\sqrt{2gh}$, the missing energy is the work done against friction or drag.

5 Power and efficiency (P3.7)

Two people can both climb the same stairs and gain the same GPE, yet one is more tired than the other, because she did it *faster*. **Power** measures how fast energy is transferred:

$$P = \frac{E}{t} = \frac{W}{t} \quad (\text{power} = \text{energy transferred} \div \text{time}).$$

The unit is the **watt** (W): $1 \text{ W} = 1 \text{ J/s}$. A 60 W bulb transfers 60 J every second. Larger units: $1 \text{ kW} = 1000 \text{ W}$ and $1 \text{ MW} = 10^6 \text{ W}$.



60 kg student climbs in 6 s

$$E_p = mgh = 60 \times 10 \times 3 = 1800 \text{ J}$$

$$P = \frac{E}{t} = \frac{1800}{6} = 300 \text{ W}$$

In the picture the 60 kg student gains 1800 J in 6 s, so her useful power is $1800/6 = 300 \text{ W}$. If she took twice as long, she would gain the same energy at half the power.

Power of a moving force. If a force F moves an object steadily through a distance d in time t , the work is Fd , so $P = \frac{Fd}{t} = F \times \frac{d}{t}$. But d/t is the speed v , so

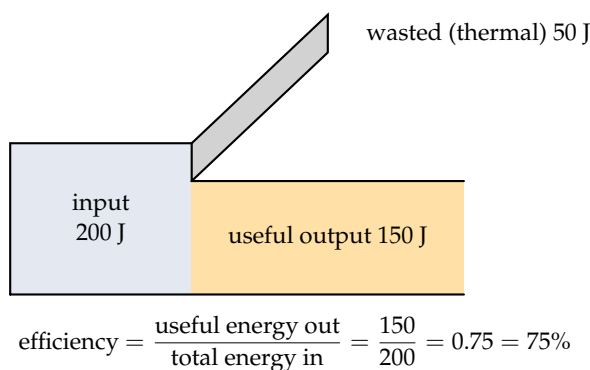
$$P = Fv.$$

This is the link between engine power, the driving force and the speed: at a steady speed, the driving force equals the resistive forces, so a car whose engine delivers 24 kW to the wheels at 20 m/s is pushing forwards with $F = P/v = 24000/20 = 1200 \text{ N}$.

Efficiency. No device turns all its input energy into the form we want: some always ends up as thermal energy in the surroundings. The **efficiency** measures how good a device is:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}.$$

It is a ratio, so it has no unit; it is written as a decimal or a percentage. Because energy is conserved, the useful output can never exceed the input, so efficiency is *always below* 100% in real devices.



The Sankey diagram draws the energy flows as arrows whose widths are proportional to the amounts. Of the 200 J going in, 150 J is useful and 50 J is wasted as thermal energy, so the efficiency is $150/200 = 0.75 = 75\%$. The arrows always add up: useful + wasted = input.

Worked example: a lifting motor

A motor draws 400 W of electrical power and lifts a 50 kg load through 12 m in 20 s. Find its efficiency.
Useful energy = $mgh = 50 \times 10 \times 12 = 6000 \text{ J}$, so useful power = $6000/20 = 300 \text{ W}$. Efficiency = $300/400 = 0.75 = 75\%$.

Worked example: engine power and force

A car's engine takes in energy at 40 kW and is 25% efficient. At a steady 20 m/s, what is the total resistive force on the car?
Useful power = $0.25 \times 40\,000 = 10\,000 \text{ W}$. Steady speed means driving force = resistive force, and $F = P/v = 10000/20 = 500 \text{ N}$.

Common mistakes

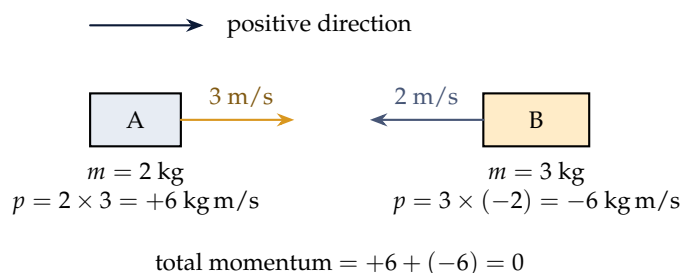
- Mixing up input and output: efficiency is *useful out* $\div$ *total in*, never the other way up.
- Giving an efficiency above 100%: a sign that you divided the wrong way round.
- Calling a kilowatt-hour a unit of power. It is a unit of *energy*: $1 \text{ kW} \times 1 \text{ h} = 3.6 \times 10^6 \text{ J}$.

6 Momentum and its conservation (P3.6)

A heavy lorry at 10 m/s is much harder to stop than a bicycle at the same speed, and a bicycle at 10 m/s is harder to stop than the same bicycle at 1 m/s. Both mass and velocity matter, so we define **momentum** as their product:

$$p = mv \quad (\text{momentum} = \text{mass} \times \text{velocity}),$$

measured in kg m/s. Momentum is a **vector**: it points in the direction of the velocity. In one dimension we show direction with a sign. Choose one direction to be positive; motion that way has positive momentum, and motion the opposite way has negative momentum.



Object A has $p = 2 \times 3 = +6$ and B (heavier but moving the other way) has $p = 3 \times (-2) = -6$. The **total** momentum is the sum, taking signs into account, here $+6 + (-6) = 0$.

Conservation of momentum. In any collision or explosion in which no external force acts on the objects (a closed system), the *total momentum before equals the total momentum after*. Why? When A pushes B, Newton's third law says B pushes

A with an equal and opposite force; the two forces last for exactly the same time. So the push changes B's momentum by some amount and A's momentum by exactly the opposite amount, and the total is unchanged. (We shall make this precise in section 8.)

The method for any problem: (1) choose a positive direction; (2) write the total momentum before, with signs; (3) write the total after; (4) equate them and solve.

Worked example: momentum of a car

Find the momentum of a 1200 kg car moving at 15 m/s. What is the momentum of the same car driving back the other way at the same speed?

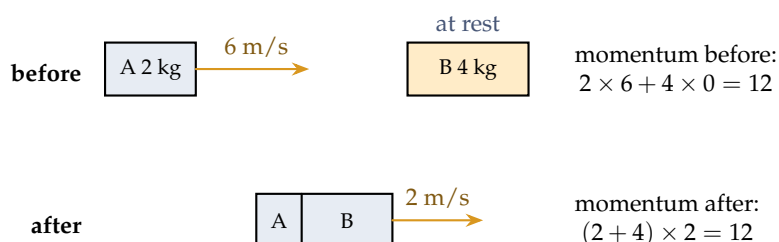
$p = mv = 1200 \times 15 = 18\,000 \text{ kg} \cdot \text{m/s}$. Reversed, the momentum is $-18\,000$: same size, opposite direction.

Exam tip

Momentum is not energy. Momentum can be positive or negative and is always conserved in a closed system. Kinetic energy is never negative and may or may not be conserved (next section). They are linked by $E_k = \frac{p^2}{2m}$, which follows from $\frac{1}{2}mv^2 = \frac{(mv)^2}{2m}$. So for the *same momentum*, the lighter object carries more KE.

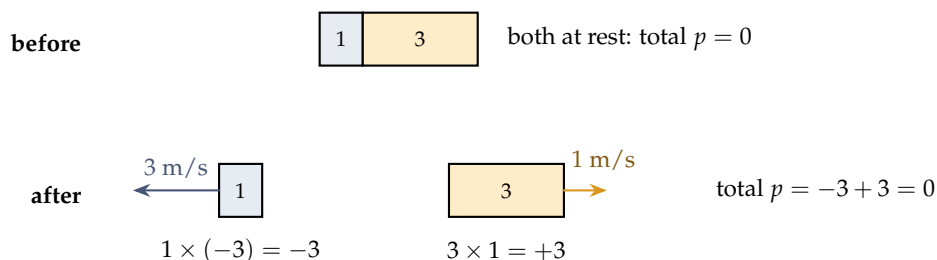
7 Collisions and explosions (P3.6)

Sticking together. In the diagram a 2 kg trolley moving at 6 m/s hits a stationary 4 kg trolley, and they stick.



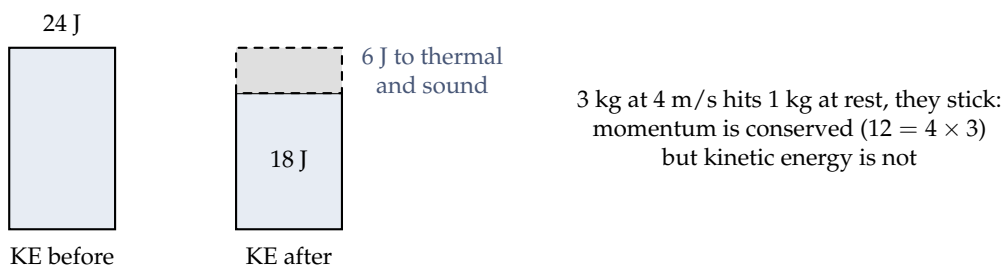
Before: $2 \times 6 + 4 \times 0 = 12 \text{ kg} \cdot \text{m/s}$. After, the combined mass $2 + 4 = 6 \text{ kg}$ moves at some speed v with momentum $6v$. Equating, $6v = 12$, so $v = 2 \text{ m/s}$. The heavier trolley has slowed the lighter one a lot, and the pair moves slowly.

Explosions and recoil. When two stationary objects push each other apart (a spring-loaded trolley, a gun firing, a rocket expelling gas), the total momentum before is zero, so the total after is still zero: the two momenta are equal and opposite.



The 1 kg piece goes left at 3 m/s ($p = -3$) and the 3 kg piece goes right at 1 m/s ($p = +3$): the lighter piece moves *faster*, in the opposite direction. This is why a gun recoils (slowly, because it is heavy) as the bullet flies off (fast, because it is light).

Is kinetic energy conserved? Momentum is conserved in every closed collision. Kinetic energy is not, unless the collision is **elastic**, meaning the objects bounce apart and no energy is transferred to thermal energy or sound. If the objects deform, stick together or make a noise, the collision is **inelastic** and some KE is lost. Two objects sticking together is the most inelastic case.



In the diagram a 3 kg trolley at 4 m/s ($E_k = \frac{1}{2} \times 3 \times 16 = 24\text{ J}$) sticks to a stationary 1 kg trolley. Momentum: $3 \times 4 = 4v$, so $v = 3\text{ m/s}$, and the new KE is $\frac{1}{2} \times 4 \times 9 = 18\text{ J}$. Momentum has been conserved (12 before and after), but 6 J of kinetic energy has become thermal energy and sound. Check: in any inelastic collision the KE afterwards is smaller than before, never larger.

Worked example: head-on and stuck

A 3 kg ball moving right at 4 m/s hits a 1 kg ball moving left at 4 m/s, and they stick. Find the speed and direction afterwards, and the KE lost.

Take right as positive. Before: $3 \times 4 + 1 \times (-4) = 8$. After: $4v = 8$, so $v = +2\text{ m/s}$ (to the right). KE before $= \frac{1}{2} \times 3 \times 16 + \frac{1}{2} \times 1 \times 16 = 24 + 8 = 32\text{ J}$. KE after $= \frac{1}{2} \times 4 \times 4 = 8\text{ J}$. KE lost $= 24\text{ J}$.

Common mistakes

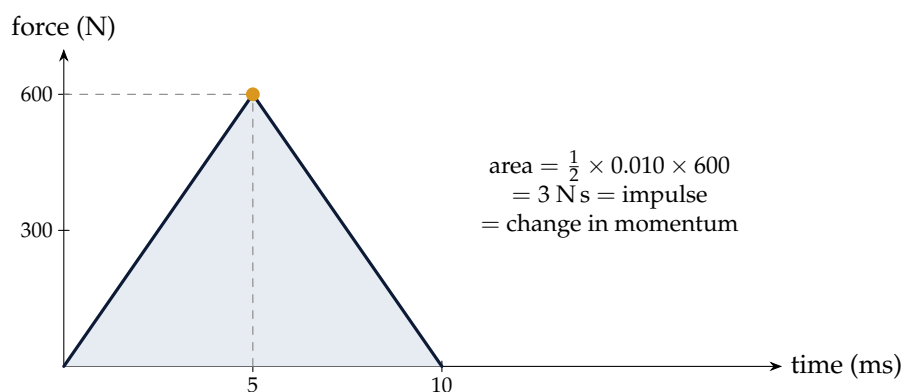
- Ignoring signs: two objects moving towards each other have momenta of *opposite* signs, so you subtract.
- Assuming kinetic energy is conserved because momentum is. In sticking collisions it is not.
- Adding the speeds instead of the momenta, or forgetting to include the stationary object's mass in the combined mass after sticking.

8 Force, impulse and the rate of change of momentum (P3.6)

Newton's second law, $F = ma$, can be written another way that is far more general. Acceleration is the change in velocity per second, $a = \frac{v - u}{t}$, so

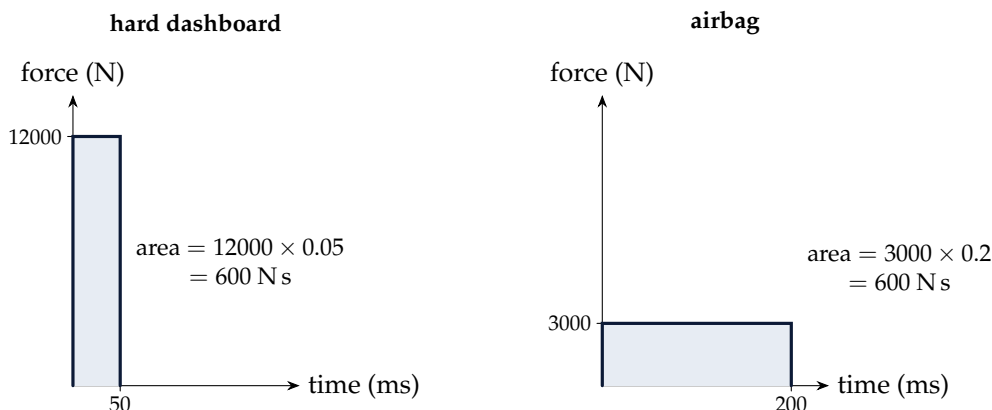
$$F = ma = \frac{m(v - u)}{t} = \frac{mv - mu}{t} = \frac{\Delta p}{t}.$$

The resultant force equals the rate of change of momentum. Rearranged, $Ft = \Delta p$: the product Ft is called the **impulse**, and it equals the change in momentum. A graph of force against time therefore has an area equal to the impulse (just as a force–distance graph's area is work).



A bat strikes a ball and the force rises to 600 N and falls again, all within 10 ms. The area is $\frac{1}{2} \times 0.010 \times 600 = 3\text{ N.s}$, so the ball's momentum changes by 3 kg.m/s . If the ball's mass is 0.15 kg and it started at rest, its final speed is $3/0.15 = 20\text{ m/s}$.

Spreading the change over a longer time. The same change in momentum can be produced by a large force for a short time or a small force for a long time, since $F = \Delta p/t$. This is the principle behind every piece of safety equipment.



A 60 kg passenger at 10 m/s has momentum $600 \text{ kg} \cdot \text{m/s}$, and all of it must be removed in a crash. Hitting a hard dashboard and stopping in 0.05 s takes an average force of $600/0.05 = 12000 \text{ N}$. An airbag lets the body stop over 0.2 s, needing only $600/0.2 = 3000 \text{ N}$. The areas are equal (600 N s); only the force is reduced. The same reasoning explains crumple zones, cycle helmets, padded playgrounds, and why you bend your knees when you land from a jump.

Worked example: crash force

A 75 kg driver travelling at 12 m/s is brought to rest by a seat belt in 0.30 s. What is the average force on the driver?

$$\Delta p = 75 \times 12 = 900 \text{ kg} \cdot \text{m/s}, \text{ so } F = \frac{\Delta p}{t} = \frac{900}{0.30} = 3000 \text{ N}.$$

Worked example: bouncing off a wall

A 0.5 kg ball hits a wall at 8 m/s and rebounds at 6 m/s. The contact lasts 0.01 s. Find the average force.

Take towards the wall as positive. $\Delta p = 0.5 \times (-6) - 0.5 \times 8 = -3 - 4 = -7$, so $|\Delta p| = 7 \text{ N} \cdot \text{s}$ and $F = 7/0.01 = 700 \text{ N}$. The change in momentum is *larger* than the momentum was, because the velocity reversed: it is $m(v - u)$ with signs.

Common mistakes

- Using $F = mv/t$ and forgetting that it is the *change* in velocity that counts; for a rebound, add the sizes of the two speeds.
- Thinking an airbag reduces the momentum change. It does not: it increases the time, which reduces the force.
- Mixing units: the time must be in seconds ($10 \text{ ms} = 0.010 \text{ s}$).

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. How much work is done by a 25 N force moving an object 8 m in the direction of the force?
2. A 4 kg mass is raised 3 m and moves at 5 m/s at the top. Find its GPE gain and its KE.
3. A car's speed rises from 10 m/s to 30 m/s. By what factor does its KE increase?
4. A stone falls from rest through 45 m (ignore air resistance). What is its speed on landing?
5. A 2 kW heater runs for 3 min. How much energy does it transfer?
6. A machine takes in 500 J and wastes 125 J. What is its efficiency?
7. A 1 kg trolley at 3 m/s hits a stationary 2 kg trolley and they stick. What is their speed?
8. A 0.4 kg ball is kicked from rest and leaves the foot at 15 m/s after a contact time of 0.02 s. What is the average force?

Answers to the self-check

1. $W = Fd = 25 \times 8 = 200 \text{ J}$.
2. $\text{GPE} = 4 \times 10 \times 3 = 120 \text{ J}$. $\text{KE} = \frac{1}{2} \times 4 \times 25 = 50 \text{ J}$.
3. $\text{KE} \propto v^2$, so the factor is $(30/10)^2 = 9$.
4. $v^2 = 2gh = 2 \times 10 \times 45 = 900$, so $v = 30 \text{ m/s}$.
5. $E = Pt = 2000 \times 180 = 360\,000 \text{ J}$.
6. $\text{Useful} = 500 - 125 = 375$, so $\text{efficiency} = 375/500 = 0.75 = 75\%$.
7. $1 \times 3 = (1 + 2)v$, so $v = 1 \text{ m/s}$.
8. $\Delta p = 0.4 \times 15 = 6 \text{ kg} \cdot \text{m/s}$, so $F = 6/0.02 = 300 \text{ N}$.

Next step

Heurisco's *ESAT Physics: Momentum, Energy and Power Practice Set* has twenty original exam-style questions on this lesson, with diagrams and graphs, in order of difficulty, with a full worked solution and a note on the trap in each.