

TMUA

Paper 1

Special Edition

Mock paper: sit it in one go, under timed conditions.

Questions	20
Time allowed	75 minutes
Calculator	Not allowed
Marking	1 mark per question, no penalty

Instructions

- Answer **all 20 questions**. Each has exactly one correct option.
- You have **75 minutes**. That is about 225 seconds a question, so if you are stuck, move on and return if time allows.
- Each correct answer scores 1 mark. There is no penalty for a wrong or blank answer, so always give an answer.
- You **must not** use a calculator. Use the blank space for rough working.
- Write your answers on the answer grid at the end, or on paper. Mark the paper afterwards with the separate answer key and worked solutions.

Sit this paper in one go, with a timer, in a quiet place. Do not look at the solutions until you have finished. After marking, read the worked solution for every question you missed *and* every question you guessed.

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- 1 For how many integers k is $x^2 - 2kx + k + 2 \geq 0$ true for *every* real number $x \geq 0$?
- A 10
- B 8
- C 7
- D 6
- E 5
- F 4
-
- 2 For how many integers k does the equation $4^x - 2^{x+2} = k$ have exactly two distinct real solutions?
- A 7
- B 6
- C 5
- D 4
- E 3
- F 2

3 How many real numbers x with $0 \leq x \leq 2\pi$ satisfy $\tan^2 x = 3 \sin x$?

A 6

B 5

C 4

D 3

E 2

F 1

4 Let $S_n = \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{n(n+2)}$. What is the smallest positive integer n for which $S_n > \frac{37}{50}$?

A 95

B 96

C 97

D 98

E 99

F 100

- 5 For each real number a consider the equation $(x - a)(x^2 - 3x + a) = 0$. What is the sum of all the values of a for which the equation has exactly two distinct real solutions?
- A 6
- B $\frac{9}{2}$
- C $\frac{17}{4}$
- D 4
- E $\frac{9}{4}$
- F 2
-
- 6 The function f is defined for all real x by $f(x) = x^3 + x^2 + x$. It is increasing, so it has an inverse f^{-1} . How many points do the graphs of $y = f(x)$ and $y = f^{-1}(x)$ have in common?
- A 2
- B 3
- C 4
- D 5
- E 6
- F 7

- 7 ABC is an equilateral triangle with side 2. Three circular arcs are drawn, each bulging outwards from the triangle: one with centre A joining B to C , one with centre B joining C to A , and one with centre C joining A to B . They bound a rounded, triangle-like shape. What is the area of the region bounded by the three arcs?
- A 2π
- B $3\pi - 2\sqrt{3}$
- C $2\pi - \sqrt{3}$
- D $2\sqrt{3}$
- E $2\pi - 2\sqrt{3}$
- F $2\pi - 3\sqrt{3}$
-
- 8 P is the point $(1, 1)$ on the curve $y = x^3$. The tangent to the curve at P meets the curve again at Q , and the tangent at Q meets the curve again at R . What is the area of triangle PQR ?
- A 27
- B $\frac{81}{2}$
- C 54
- D 81
- E 108
- F 162

- 9 The line $y = mx + 2$, where $m > 0$, meets the parabola $y = x^2$ at two points. The finite region between the line and the parabola has area 36. What is the value of m ?
- A $\sqrt{7}$
- B 4
- C $2\sqrt{7}$
- D $4\sqrt{2}$
- E 6
- F 8
-
- 10 The trapezium rule with n strips of equal width is used to estimate $\int_0^2 (x^3 - 3x^2) dx$. For which positive integers n is the estimate exactly equal to the true value of the integral?
- A every positive integer n
- B no value of n
- C $n = 1$ only
- D even n only
- E $n = 1$ and $n = 2$ only
- F odd n only

- 11** Consider $y = \left| |x^2 - 4x| - 3 \right|$. For each real number k , let $N(k)$ be the number of real solutions of $\left| |x^2 - 4x| - 3 \right| = k$. What is the value of $N(0) + N(1) + N(2) + N(3) + N(4)$?
- A** 26
- B** 24
- C** 23
- D** 22
- E** 21
- F** 20
-
- 12** The points $A(-2, 0)$ and $B(2, 0)$ are fixed. For each real number c , consider the points P on the line $y = x + c$ for which angle APB is exactly 90° . For how many values of c is there exactly one such point P ?
- A** 8
- B** 6
- C** 5
- D** 4
- E** 3
- F** 2

13 How many real numbers x with $0 \leq x < 2\pi$ satisfy $\cot x + \cot^2 x + \cot^3 x + \cdots = 2 \tan x$, where the left-hand side is the sum of the infinite series?

A 8

B 6

C 5

D 4

E 3

F 2

14 A sequence is defined by $u_1 = 3$ and $u_{n+1} = u_n^2 - 2u_n + 2$ for $n \geq 1$. What is the units digit of u_{2025} ?

A 9

B 7

C 6

D 5

E 3

F 1

- 15** The point $P(3, 1)$ lies inside the circle $x^2 + y^2 = 25$. A *chord* of the circle is a line segment joining two points of the circle. How many chords of the circle that pass through P have an integer length?
- A** 8
- B** 7
- C** 6
- D** 5
- E** 4
- F** 3
-
- 16** Two perpendicular lines meet at a point O . The circle C_0 of radius 1 touches both lines. The circle C_1 touches both lines and touches C_0 , and lies between C_0 and O . The circle C_2 touches both lines and C_1 , and lies between C_1 and O , and so on for ever. What is the total area of all the circles $C_0, C_1, C_2, \dots$?
- A** π
- B** $\frac{\pi(4 + 3\sqrt{2})}{8}$
- C** $\frac{9\pi}{8}$
- D** $\frac{\pi(1 + \sqrt{2})}{2}$
- E** $\frac{4\pi}{3}$
- F** $\pi(1 + \sqrt{2})$

- 17 For a constant $q > 1$, the curve $y = x(x - 1)(x - q)$ and the x -axis enclose two finite regions. The two regions have equal area. What is the area of one of them?

- A 1
- B $\frac{3}{4}$
- C $\frac{1}{2}$
- D $\frac{5}{12}$
- E $\frac{1}{3}$
- F $\frac{1}{4}$

- 18 The function f is defined by $f(x) = x^3 + x$ for all real x . It is increasing, so it has an inverse function f^{-1} . What is the value of $\int_0^2 f^{-1}(x) dx$?

- A $\frac{3}{4}$
- B 1
- C $\frac{5}{4}$
- D $\frac{3}{2}$
- E $\frac{7}{4}$
- F 2

19 What is the area of the region of the plane defined by $|x| + |y| + |x - y| \leq 4$?

A 32

B 24

C 20

D 16

E 14

F 12

20 The curve C has equation $y = x^3 - 3x$. For certain real numbers k , exactly two different tangent lines to C pass through the point $(2, k)$. What is the sum of all such values of k ?

A -6

B -4

C -2

D 0

E 2

F 4

Answer grid

1 ____ 2 ____ 3 ____ 4 ____ 5 ____ 6 ____ 7 ____ 8 ____ 9 ____
10 ____ 11 ____ 12 ____ 13 ____ 14 ____ 15 ____ 16 ____ 17 ____ 18 ____
19 ____ 20 ____