

TMUA

Paper 2

Special Edition Solutions

Answer key, worked solutions and score guide.

Questions	20
Time allowed	75 minutes
Calculator	Not allowed
Marking	1 mark per question, no penalty

Answer key

1 C 2 B 3 E 4 A 5 D 6 G 7 E 8 D 9 C
 10 B 11 G 12 F 13 F 14 C 15 D 16 C 17 G 18 E
 19 F 20 A

Specification codes tested: 2.1, 2.2, 2.5.

Score guide

This table is a rough planning aid. It is **not** an official score: UAT-UK does not publish a conversion for our papers, and real conversions change with every sitting.

Score	Percentage	Suggested next step
0–6	0–30%	Go back to the free lessons for this module, then try the topic sets.
7–10	35–50%	Fix the topics you missed with topic sets, then retake a different mock.
11–14	55–70%	Solid base. Work on speed and the questions you guessed.
15–17	75–85%	Strong. Practise full-length papers back to back.
18–20	90–100%	Excellent. Check for careless slips, then keep timing sharp.

Worked solutions

1. Answer C

2.5

Question: How many ordered pairs (a, b) of positive integers satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}$?

Clear denominators: $6a + 6b = ab$, i.e. $ab - 6a - 6b = 0$. Add 36 to both sides: $(a - 6)(b - 6) = 36$.

Since $\frac{1}{a} < \frac{1}{6}$ we need $a > 6$, and likewise $b > 6$, so both factors $a - 6$ and $b - 6$ are *positive* integers with product 36. (Negative pairs would need, say, $a - 6 \in \{-1, \dots, -5\}$, forcing $b - 6 \leq -8$, i.e. $b < 0$.) The positive divisors of $36 = 2^2 \cdot 3^2$ number 9, and each gives one ordered pair: $(a - 6, b - 6) = (1, 36), (2, 18), (3, 12), (4, 9), (6, 6), (9, 4), (12, 3), (18, 2), (36, 1)$. So 9.

Trap. 5: counting unordered pairs, which merges (a, b) and (b, a) although the question asks for ordered pairs; or 10: counting both signs or the extra factor pair $(a - 6)(b - 6) = 36$ with negative factors.

2. Answer B

2.5

Question: Let a and b be positive integers. Which of the following statements are true for all such a and b ?

- I If a^2 divides b^2 , then a divides b .
- II If a^2 divides b^3 , then a divides b .
- III If a^3 divides b^4 , then a divides b .

Compare exponents of a prime p : say $p^\alpha \parallel a$ and $p^\beta \parallel b$. Then $a^m \mid b^n$ for all primes means $m\alpha \leq n\beta$, while $a \mid b$ means $\alpha \leq \beta$.

I. $2\alpha \leq 2\beta$ gives $\alpha \leq \beta$: true.

II. $2\alpha \leq 3\beta$ only gives $\alpha \leq 1.5\beta$. Try $\alpha = 3, \beta = 2$: $a = 8, b = 4$ gives $a^2 = 64 = b^3$, yet $8 \nmid 4$. False.

III. $3\alpha \leq 4\beta$ allows $\alpha = 4, \beta = 3$: $a = 16, b = 8$ gives $a^3 = 4096 = b^4$ but $16 \nmid 8$. False. So I only.

Trap. "I and II only" or "I, II and III": the square case I is true, and the step from " $a^2 \mid b^2$ " to " $a \mid b$ " is wrongly assumed to survive when the exponents on the two sides differ.

3. Answer E

2.2

Question: A mathematician has invented a strange way of adding real numbers: it writes $x * y$ for $x + y + xy$. Which of the following statements are true?

- I $(x * y) * z = x * (y * z)$ for all real x, y, z .
- II There is exactly one real number e such that $e * x = x$ for every real number x .
- III For every real number x there is a real number y such that $x * y = 0$.

The key is $x * y = (x + 1)(y + 1) - 1$.

I. $(x * y) * z + 1 = (x + 1)(y + 1)(z + 1) = x * (y * z) + 1$. True.

II. $e * x = x$ for all x means $(e + 1)(x + 1) = x + 1$, so $e(x + 1) = 0$ for all x ; taking $x = 0$ gives $e = 0$, and $e = 0$ works. So exactly one such e : True.

III. $x * y = 0$ means $(x + 1)(y + 1) = 1$. For $x = -1$ the left side is 0 for every y : no y exists. False. So I and II only.

Trap. "I, II and III": the inverse $y = -\frac{x}{x+1}$ exists for every x except the single value $x = -1$, which is exactly the case that breaks the statement.

4. Answer A

2.1

Question: For $r > 0$, let C_r be the circle with centre $(0, r)$ and radius r ; it touches the x -axis at the origin O . Let $P(r)$ be the statement “ C_r and the parabola $y = x^2$ have no common point other than O ”. Which of the following is true?

A common point (x, y) has $y = x^2$ and $x^2 + (y - r)^2 = r^2$. Substituting, $x^2 + (x^2 - r)^2 = r^2$, which simplifies to $x^2(x^2 + 1 - 2r) = 0$.

So besides $x = 0$ (the point O) there are further common points exactly when $x^2 = 2r - 1$ has a solution, that is, when $r > \frac{1}{2}$. Hence $P(r)$ holds exactly when $r \leq \frac{1}{2}$.

Therefore $r < \frac{1}{2}$ is sufficient. It is not necessary, because $r = \frac{1}{2}$ also satisfies P : then the equation is $x^4 = 0$, and the circle merely hugs the parabola at O with no other contact.

Trap. “Necessary and sufficient”: the quartic has a root $x^2 = 2r - 1$ only for $r > \frac{1}{2}$ and the borderline case $r = \frac{1}{2}$ (a quadruple root at O) is easily dropped.

5. Answer D

2.5

Question: Six creators play a round-robin collab: every pair plays exactly one game, which is always won by one of the two (no draws), and each win scores one point. Which of the following lists could be the six final scores, in some order?

I 5, 5, 2, 1, 1, 1

II 5, 4, 4, 1, 1, 0

III 4, 4, 4, 1, 1, 1

All lists add to 15, the number of games, so the total does not decide.

I. A score of 5 means beating all five others, including the other player with 5: impossible for two players. False.

II. The 5 beats everyone, so each of the two players with 4 lost to him and must have beaten all the other four players, including each other: impossible. False.

III. Let X, Y, Z beat each of the three others U, V, W , and let X beat Y , Y beat Z , Z beat X ; let U beat V , V beat W , W beat U . Then X, Y, Z score $3 + 1 = 4$ each and U, V, W score 1 each. True. So III only.

Trap. “I, II and III” or “III only” mis-read: since each list has the right total (15) it is tempting to accept them all.

6. Answer G

2.5

Question: Let f be a function from the real numbers to the real numbers such that, for every real x ,

$$f(x) = f(2 - x) \quad \text{and} \quad f(-x) = -f(x).$$

Which of the following must be true?

I $f(2025) = 0$.

II $f(x + 4) = f(x)$ for every real x .

III $f(2026) = 0$.

Combine the two symmetries: $f(x + 2) = f(2 - (x + 2)) = f(-x) = -f(x)$. Applying this twice, $f(x + 4) = -f(x + 2) = f(x)$: **II is true**, and the graph repeats every 4.

Putting $x = 0$ in $f(-x) = -f(x)$ gives $f(0) = 0$, and then $f(2) = f(2 - 2) = f(0) = 0$. Since $2026 = 2 + 4 \times 506$, $f(2026) = f(2) = 0$: **III is true**.

For I, $2025 = 1 + 4 \times 506$ gives $f(2025) = f(1)$, and nothing forces $f(1) = 0$: $f(x) = \sin \frac{\pi x}{2}$ satisfies both conditions and has $f(1) = 1$. **I is false**. So II and III only.

Trap. “I, II and III”: noticing the period 4 and then assuming the odd symmetry forces zeros at every integer; it forces them only at the even ones.

7. Answer E

2.5

Question: A baker writes the numbers $1, 2, 3, \dots, n$ on an order board. In each step they choose two numbers on the board, wipe them, and write their (positive or zero) difference. After $n - 1$ steps one number is left. Which of the following statements are true?

- I For $n = 8$, the final number can be 0.
- II For $n = 9$, the final number can be 9.
- III For $n = 10$, the final number can be 0.

Invariant. Replacing a, b by $|a - b|$ changes the total by $a + b - |a - b| = 2 \min(a, b)$, an even number, so the parity of the total never changes. The total starts at $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ and the final number has that parity.

III is false: for $n = 10$ the total is 55, odd, so the final number is odd and cannot be 0.

I is true. The total 36 is even, and parity alone does not guarantee a construction, so build one: $(8, 7) \rightarrow 1$, $(6, 5) \rightarrow 1$, $(4, 3) \rightarrow 1$, $(2, 1) \rightarrow 1$, leaving $1, 1, 1, 1$, then $(1, 1) \rightarrow 0$ twice, leaving $0, 0$ and finally 0.

II is true. The total 45 is odd, so 9 is not ruled out; build it: $(1, 2) \rightarrow 1$, $(3, 4) \rightarrow 1$, $(5, 6) \rightarrow 1$, $(7, 8) \rightarrow 1$ leaves $9, 1, 1, 1, 1$, then $(1, 1) \rightarrow 0$ twice gives $9, 0, 0$ and finally 9. So I and II only.

Trap. “I only” or “I and III only”: the parity test is only a *necessary* condition; it rules out III and $n = 9$ ending in 0 but, for II, one must check that the odd value 9 is actually reachable (it is).

8. Answer D

2.5

Question: In triangle ABC , M is the midpoint of BC and G is the centroid (the point where the three medians meet). Which of the following statements are true?

- I Every straight line that divides the triangle into two parts of equal area passes through G .
- II Every straight line through G divides the triangle into two parts of equal area.
- III Every straight line through M that divides the triangle into two parts of equal area passes through A .

I is false. Let a line parallel to BC cut off a triangle at the top similar to ABC ; its area is half when the scale factor is $\frac{1}{\sqrt{2}}$, so the line is at distance $\left(1 - \frac{1}{\sqrt{2}}\right)h \approx 0.29h$ above BC (h the height). But G is at height $\frac{1}{3}h$, so this line misses G .

II is false. The line through G parallel to BC cuts off a top triangle of scale $\frac{2}{3}$, area $\frac{4}{9}$ of the whole, and leaves $\frac{5}{9}$.

III is true. A line through M other than BC meets side AB (say) at P . The triangle BMP has area $\frac{BM}{BC} \cdot \frac{BP}{BA} = \frac{1}{2} \cdot \frac{BP}{BA}$ of the whole (the area is proportional to the product of the two sides about the common angle B). That equals $\frac{1}{2}$ only when $BP = BA$, i.e. $P = A$. So the only such line is the median MA . So III only.

Trap. “II and III only” or “I and III only”: the medians do bisect the area and pass through G , which makes “through $G \Leftrightarrow$ bisects” look plausible in both directions; the parallel lines in the proof break each direction.

9. Answer C

2.5

Question: Which of the following equations has at least one solution in integers x, y (and z)?

I $x^2 + y^2 = 2023$

II $x^2 - y^2 = 2024$

III $x^2 + y^2 + z^2 = 2031$

I. A square leaves remainder 0 or 1 on division by 4, so $x^2 + y^2$ leaves 0, 1 or 2. But $2023 = 4 \times 505 + 3$. No solution.

II. $x^2 - y^2 = (x - y)(x + y)$. Take $x - y = 2$ and $x + y = 1012$ (same parity, product 2024): $x = 507, y = 505$. A solution exists.

III. A square leaves remainder 0, 1 or 4 on division by 8 (odd squares leave 1, and $(2k)^2$ leaves 0 or 4). Three of these add up to at most remainders 0, 1, 2, 3, 4, 5, 6; never 7 (try the cases: three terms from $\{0, 1, 4\}$ give 0, 1, 2, 3, 4, 5, 6, 8, 9, 12, i.e. remainders 0 to 6). Since $2031 = 8 \times 253 + 7$, there is no solution. So II only.

Trap. “II and III only” or “I, II and III”: trying to find a solution for I or III by searching, instead of testing the remainders on division by 4 and 8; a failed search proves nothing, a failed remainder test does.

10. Answer B

2.5

Question: How many integers x with $0 \leq x < 100$ are there for which $x^2 - x$ is a multiple of 100?

$x^2 - x = x(x - 1)$, and x and $x - 1$ share no common factor greater than 1. For $100 = 4 \times 25$ to divide the product, each of the coprime pieces 4 and 25 must divide *one* of the two factors: $4 \mid x$ or $4 \mid (x - 1)$, and $25 \mid x$ or $25 \mid (x - 1)$.

That gives four combinations. $4 \mid x, 25 \mid x$: $x = 0$. $4 \mid x - 1, 25 \mid x - 1$: $x = 1$. $25 \mid x, 4 \mid x - 1$: $x \in \{0, 25, 50, 75\}$ with $x \equiv 1 \pmod{4}$: $x = 25$. $4 \mid x, 25 \mid x - 1$: $x \in \{1, 26, 51, 76\}$ with $4 \mid x$: $x = 76$. So $\{0, 1, 25, 76\}$: 4 values.

Trap. 2: noticing that $x = 0$ and $x = 1$ work and assuming a product of consecutive numbers must be divisible by 100 only when one factor is; the coprime pieces 4 and 25 can be split between the two factors.

11. Answer G

2.1

Question: A counter starts at the point $(0, 0)$ of the integer lattice. In each move it goes either 1 step right and 2 steps up, or 2 steps right and 1 step up. For integers $x, y \geq 0$, which of the following are necessary *and* sufficient for the counter to be able to reach (x, y) after some number (possibly zero) of moves?

I $x + y$ is a multiple of 3.

II $x + y$ is a multiple of 3, and $x \leq 2y$ and $y \leq 2x$.

III $2x - y$ and $2y - x$ are both non-negative multiples of 3.

After a moves of the first kind and b of the second, the counter is at $(a + 2b, 2a + b)$ with $a, b \geq 0$ integers. Solving, $a = \frac{2y - x}{3}$ and $b = \frac{2x - y}{3}$. So (x, y) is reachable exactly when $2y - x$ and $2x - y$ are non-negative multiples of 3: **III is necessary and sufficient.**

Since $2x - y \equiv 2(x + y) \equiv 2y - x \pmod{3}$, both are multiples of 3 exactly when $3 \mid x + y$, and they are non-negative exactly when $x \leq 2y$ and $y \leq 2x$: so **II is the same condition.** I is necessary but not sufficient: $(3, 0)$ has $x + y = 3$ but is unreachable. So II and III only.

Trap. “I and II only” or “I only”: finding the invariant $x + y \equiv 0 \pmod{3}$ and stopping, without noticing that the moves can only point into the cone $\frac{1}{2}y \leq x \leq 2y$.

12. Answer F

2.5

Question: Which of the following statements are true?

- I For every integer $n > 1$, $n^4 + 64$ is not prime.
- II For every integer $n > 1$, $n^4 + 16$ is not prime.
- III For every integer $n > 1$, $n^4 + n^2 + 1$ is not prime.

I and III come from a difference of two squares. $n^4 + 64 = (n^2 + 8)^2 - 16n^2 = (n^2 + 4n + 8)(n^2 - 4n + 8)$, and for $n > 1$ both factors exceed 1 (the smaller is $(n - 2)^2 + 4 \geq 4$, and the larger exceeds it). Likewise $n^4 + n^2 + 1 = (n^2 + 1)^2 - n^2 = (n^2 + n + 1)(n^2 - n + 1)$ with $n^2 - n + 1 \geq 3$ and $n^2 + n + 1 > n^2 - n + 1$. So I and III are true.

II cannot be rescued by the same trick: $n = 2$ gives 32 (not prime), which tempts one to believe it, but $n = 3$ gives $81 + 16 = 97$, which is prime. So II is false. Answer: I and III only.

Trap. “I, II and III”: testing $n = 2$ (80, 32, 21, all composite) and believing the pattern; the single value $n = 3$ breaks II.

13. Answer F

2.5

Question: Points are placed anywhere in a square of side 1 or in a regular hexagon of side 1 (boundary included). Which of the following statements are true?

- I If 5 points are placed in the square, two of them are at most $\frac{\sqrt{2}}{2}$ apart.
- II If 4 points are placed in the square, two of them are at most $\frac{\sqrt{2}}{2}$ apart.
- III If 7 points are placed in the hexagon, two of them are at most 1 apart.

I. Cut the square into four squares of side $\frac{1}{2}$. Five points put two in the same small square (pigeonhole), and two points in a square of side $\frac{1}{2}$ are at most its diagonal $\frac{\sqrt{2}}{2}$ apart. True.

II. The four corners of the square have all mutual distances 1 or $\sqrt{2}$, all greater than $\frac{\sqrt{2}}{2}$: a counterexample. False.

III. Join the centre of the hexagon to its vertices to cut it into six equilateral triangles of side 1. Seven points put two in the same triangle, and two points of an equilateral triangle of side 1 are at most 1 apart. True. So I and III only.

Trap. “I, II and III”: after the pigeonhole argument for five points it is natural to believe four points are enough too; the four corners show that they are not.

14. Answer C

2.5

Question: For a positive integer n , the number $n! = 1 \times 2 \times \cdots \times n$ ends in some number of zeros (for example $10! = 3628800$ ends in two zeros). For how many integers k with $1 \leq k \leq 30$ is there a positive integer n such that $n!$ ends in exactly k zeros?

The zeros come from factors $10 = 2 \times 5$, and there are always more 2s than 5s, so the number of zeros is the number of factors 5 in $n!$: $Z(n) = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{25} \right\rfloor + \left\lfloor \frac{n}{125} \right\rfloor + \cdots$.

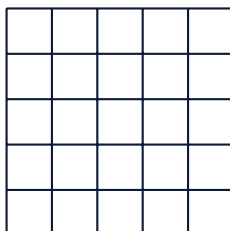
Z increases as n increases, usually by 1 when n passes a multiple of 5, but by 2 when n passes a multiple of 25 that is not a multiple of 125 (two factors of 5 arrive together), and by 3 at 125. So some values are skipped. For example $Z(24) = 4$ but $Z(25) = 6$, so 5 is never attained; the same happens at 50, 75, 100 (skipping 11, 17, 23), and at $n = 125$ the count jumps from $Z(124) = 28$ to $Z(125) = 31$, skipping 29 and 30. So the values $k \leq 30$ that are skipped are 5, 11, 17, 23, 29, 30, six of them, and $30 - 6 = 24$ values are attained.

Trap. 30 or 29: believing that the number of trailing zeros goes up one step at a time, so every value is attained; the extra factor of 5 at each multiple of 25 makes it skip.

15. Answer D

2.5

Question: A 5×5 board of unit squares, as shown, has one square removed. Someone then tries to cover the remaining 24 squares exactly, with no overlaps and nothing sticking out, using twelve 2×1 dominoes (each covers two squares that share an edge). For how many of the 25 squares is it possible, if that square is the one removed?



Colour the board like a chessboard with the corners black: there are 13 black and 12 white squares. Every domino covers exactly one black and one white square, so after the removal the remaining squares must contain equally many of each colour. Hence the removed square must be *black*: at most 13 squares are possible.

All 13 black squares do work. Snake through the board row by row (1st row left to right, 2nd right to left, and so on) to visit all 25 squares in a path in which consecutive squares share an edge. The path starts and ends on black squares and alternates colours, so the black squares are exactly its odd-numbered squares. Removing the k -th square (k odd) leaves two stretches of $k - 1$ and $25 - k$ squares, both even, and each stretch can be covered by dominoes laid along the path two at a time. So the answer is 13.

Trap. 25 (or 24): noting that 24 is even and concluding that any removal will do; 12: counting the wrong colour, the squares that can never be removed.

16. Answer C

2.5

Question: After a meet-up, the guests compare how many hands they shook. Each pair of people shakes hands at most once, and nobody shakes their own hand. Which of the following situations could actually have happened?

- I 9 people, each of whom shook hands with exactly 3 others.
- II 7 people, each of whom shook hands with exactly 4 others.
- III 6 people, who shook hands 5, 5, 4, 3, 2, 1 times respectively.

I is impossible: each handshake is counted by two people, so the total of the individual counts is even; here it would be $9 \times 3 = 27$, odd.

II is possible: seat the seven people round a table and let each shake hands with the two people on either side and the two next beyond them. Each person has exactly 4 handshakes (the total 28 is even, as it must be).

III is impossible, even though the total 20 is even: two people each shook hands with all 5 others, so everyone, including the person with 1 handshake, shook hands with both of them, forcing at least 2 handshakes for each person. Contradiction. So II only.

Trap. "II and III only": using only the parity (even total) test, which is necessary but not sufficient, to accept III.

17. Answer G

2.5

Question: Some different integers are chosen from $\{1, 2, 3, \dots, 20\}$. Which of the following statements are true?

- I Any 10 chosen integers must include two that have no common factor greater than 1.
- II Any 11 chosen integers must include two that differ by exactly 1.
- III Any 11 chosen integers must include two such that one is a multiple of the other.

I is false. The ten even numbers $2, 4, \dots, 20$ all share the factor 2.

II is true. Split $\{1, \dots, 20\}$ into the ten pairs $\{1, 2\}, \{3, 4\}, \dots, \{19, 20\}$. Eleven chosen numbers put two in one pair, and they differ by 1.

III is true. Write each number as $2^k \times m$ with m odd. There are only ten possible odd parts $(1, 3, \dots, 19)$, so among eleven chosen numbers two have the same odd part m , say $2^j m < 2^k m$ with $j < k$; then the larger is a multiple of the smaller. (Ten numbers do not suffice: $11, \dots, 20$ contain no such pair.) So II and III only.

Trap. “I, II and III”: reasoning that ten numbers must contain two consecutive (hence coprime) ones by the pairing argument; but the pairing argument needs eleven numbers, and the ten even numbers escape.

18. Answer E

2.5

Question: Consider the graph of $y = |x^2 - 4x + 3|$. For $m > 0$, and the equation $|x^2 - 4x + 3| = m|x|$, that is, where the graph meets the V-shaped graph $y = m|x|$. What is the sum of all values of m for which this equation has exactly three distinct real solutions?

Three solutions means the V touches the graph tangentially once and crosses it twice more. Split by sign of x .

For $x > 0$: the V is $y = mx$. It meets the right-hand branches where $x^2 - 4x + 3 = \pm mx$. Tangency to the central arch $y = -(x^2 - 4x + 3)$ needs $x^2 + (m - 4)x + 3 = 0$ to have a repeated root: $(m - 4)^2 = 12$. For $x < 0$ the V is $y = -mx$ and it meets the left branch where $x^2 - 4x + 3 = -mx$, i.e. $x^2 - (4 - m)x + 3 = 0$: tangency needs $(4 - m)^2 = 12$, the same equation. So $m = 4 \pm 2\sqrt{3}$.

Check the touching points are on the correct branches: for $m = 4 - 2\sqrt{3}$ (≈ 0.54) the double root is $x = \frac{4-m}{2} = \sqrt{3} \in (1, 3)$ (on the arch), and for $m = 4 + 2\sqrt{3}$ it is $x = -\sqrt{3} < 0$ (on the left branch). In each case the rest of the graph is crossed twice (the sketch shows this), giving exactly 3 solutions, while for other m there are 2 or 4. The two values sum to 8 (Vieta on $(m - 4)^2 = 12$).

Trap. $4 - 2\sqrt{3}$: finding the tangent to the central arch, the one a sketch of the right half shows, and missing the mirror-image tangency on the left branch that the modulus on x creates.

19. Answer F

2.5

Question: Let x and y be real numbers. Which of the following statements are true?

- I If x and y are rational, and $x + y$ and xy are integers, then x and y are integers.
- II If $x + y$ and xy are integers, then x and y are integers.
- III If $x + y$ and xy are both odd integers, then x and y are irrational.

x and y are the roots of $t^2 - st + p = 0$, where $s = x + y$ and $p = xy$ are integers.

I. Let a rational root be $t = \frac{a}{b}$ in lowest terms. Then $a^2 - sab + pb^2 = 0$, so b divides a^2 ; as a, b have no common factor, $b = 1$. So the root is an integer, and so is the other ($y = s - x$). True.

II. $x = \sqrt{2}, y = -\sqrt{2}$ have sum 0 and product -2 , both integers, but are not integers. False.

III. If x were rational, I would make it an integer, so $x^2 - sx + p$ has an integer root x , but then $x^2 - sx + p \equiv x^2 - x + 1 \pmod{2}$ is odd for every integer x ($x^2 - x$ is even), so it cannot be 0. Hence x is irrational, and so is y (by symmetry; $y = s - x$ is irrational too). True. So I and III only.

Trap. “I and II only” or “I, II and III”: sum and product both being integers feels as if it pins down the numbers, but $\sqrt{2}$ and $-\sqrt{2}$ show it does not; III is the one a parity argument settles.

20. Answer A

2.5

Question: A recording lasts 2025 seconds. It is to be cut into two or more segments whose lengths, in whole seconds, are consecutive positive integers, such as 5, 6, 7, ... in order, with the lengths adding up to 2025 exactly. In how many different ways can this be done? (Two ways are different if their lists of lengths are different.)

Suppose the segments are $a, a + 1, \dots, a + k - 1$ ($k \geq 2$ of them, $a \geq 1$). Their sum is $\frac{k(2a + k - 1)}{2} = 2025$, so

$$k(2a + k - 1) = 4050 = 2 \times 3^4 \times 5^2.$$

The factors k and $m = 2a + k - 1$ have opposite parity (since $m - k = 2a - 1$ is odd), so exactly one of them is even and it takes the whole factor 2: the other is an odd divisor of 2025. Conversely, any odd divisor d of 2025 gives the pair $\{d, 4050/d\}$ of opposite parity; taking k as the smaller of the two and m the larger gives $a = \frac{m-k+1}{2}$, a positive integer. So there are as many representations (with $k \geq 1$) as odd divisors of $2025 = 3^4 \cdot 5^2$, namely $5 \times 3 = 15$.

One of these is $k = 1$ (the single segment 2025, from $d = 1$, pair $\{1, 4050\}$), which is not allowed. So the answer is $15 - 1 = 14$.

Trap. 15: counting the number of odd divisors and forgetting that $k = 1$ (one segment of 2025 seconds) is not allowed; 29 and 30: counting divisors of 4050 or 2025 in both orders.