

TMUA

Paper 2

Special Edition

Mock paper: sit it in one go, under timed conditions.

Questions	20
Time allowed	75 minutes
Calculator	Not allowed
Marking	1 mark per question, no penalty

Instructions

- Answer **all 20 questions**. Each has exactly one correct option.
- You have **75 minutes**. That is about 225 seconds a question, so if you are stuck, move on and return if time allows.
- Each correct answer scores 1 mark. There is no penalty for a wrong or blank answer, so always give an answer.
- You **must not** use a calculator. Use the blank space for rough working.
- Write your answers on the answer grid at the end, or on paper. Mark the paper afterwards with the separate answer key and worked solutions.

Sit this paper in one go, with a timer, in a quiet place. Do not look at the solutions until you have finished. After marking, read the worked solution for every question you missed *and* every question you guessed.

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- 1 How many ordered pairs (a, b) of positive integers satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}$?
- A 5
- B 8
- C 9
- D 10
- E 18
- F 36
- 2 Let a and b be positive integers. Which of the following statements are true for all such a and b ?
- I If a^2 divides b^2 , then a divides b .
- II If a^2 divides b^3 , then a divides b .
- III If a^3 divides b^4 , then a divides b .
- A None of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

- 3** A mathematician has invented a strange way of adding real numbers: it writes $x * y$ for $x + y + xy$. Which of the following statements are true?
- I $(x * y) * z = x * (y * z)$ for all real x, y, z .
 - II There is exactly one real number e such that $e * x = x$ for every real number x .
 - III For every real number x there is a real number y such that $x * y = 0$.
- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III
- 4** For $r > 0$, let C_r be the circle with centre $(0, r)$ and radius r ; it touches the x -axis at the origin O . Let $P(r)$ be the statement “ C_r and the parabola $y = x^2$ have no common point other than O ”. Which of the following is true?
- A** “ $r < \frac{1}{2}$ ” is a sufficient but not necessary condition for $P(r)$.
 - B** “ $r < \frac{1}{2}$ ” is a necessary but not sufficient condition for $P(r)$.
 - C** “ $r < \frac{1}{2}$ ” is a necessary and sufficient condition for $P(r)$.
 - D** “ $r < \frac{1}{2}$ ” is neither a necessary nor a sufficient condition for $P(r)$.

- 5** Six creators play a round-robin collab: every pair plays exactly one game, which is always won by one of the two (no draws), and each win scores one point. Which of the following lists could be the six final scores, in some order?

I 5, 5, 2, 1, 1, 1

II 5, 4, 4, 1, 1, 0

III 4, 4, 4, 1, 1, 1

A None of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

- 6 Let f be a function from the real numbers to the real numbers such that, for every real x ,

$$f(x) = f(2 - x) \quad \text{and} \quad f(-x) = -f(x).$$

Which of the following must be true?

- I $f(2025) = 0$.
 - II $f(x + 4) = f(x)$ for every real x .
 - III $f(2026) = 0$.
- A None of them
 - B I only
 - C II only
 - D III only
 - E I and II only
 - F I and III only
 - G II and III only
 - H I, II and III

- 7** A baker writes the numbers $1, 2, 3, \dots, n$ on an order board. In each step they choose two numbers on the board, wipe them, and write their (positive or zero) difference. After $n - 1$ steps one number is left. Which of the following statements are true?
- I For $n = 8$, the final number can be 0.
 - II For $n = 9$, the final number can be 9.
 - III For $n = 10$, the final number can be 0.
- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III
- 8** In triangle ABC , M is the midpoint of BC and G is the centroid (the point where the three medians meet). Which of the following statements are true?
- I Every straight line that divides the triangle into two parts of equal area passes through G .
 - II Every straight line through G divides the triangle into two parts of equal area.
 - III Every straight line through M that divides the triangle into two parts of equal area passes through A .
- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

9 Which of the following equations has at least one solution in integers x , y (and z)?

I $x^2 + y^2 = 2023$

II $x^2 - y^2 = 2024$

III $x^2 + y^2 + z^2 = 2031$

A None of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

10 How many integers x with $0 \leq x < 100$ are there for which $x^2 - x$ is a multiple of 100?

A 2

B 4

C 5

D 6

E 8

F 10

- 11** A counter starts at the point $(0,0)$ of the integer lattice. In each move it goes either 1 step right and 2 steps up, or 2 steps right and 1 step up. For integers $x, y \geq 0$, which of the following are necessary *and* sufficient for the counter to be able to reach (x, y) after some number (possibly zero) of moves?

I $x + y$ is a multiple of 3.

II $x + y$ is a multiple of 3, and $x \leq 2y$ and $y \leq 2x$.

III $2x - y$ and $2y - x$ are both non-negative multiples of 3.

A None of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

- 12** Which of the following statements are true?

I For every integer $n > 1$, $n^4 + 64$ is not prime.

II For every integer $n > 1$, $n^4 + 16$ is not prime.

III For every integer $n > 1$, $n^4 + n^2 + 1$ is not prime.

A None of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

- 13** Points are placed anywhere in a square of side 1 or in a regular hexagon of side 1 (boundary included). Which of the following statements are true?

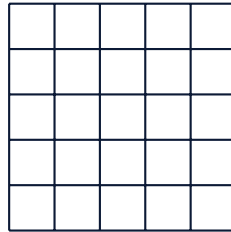
I If 5 points are placed in the square, two of them are at most $\frac{\sqrt{2}}{2}$ apart.

II If 4 points are placed in the square, two of them are at most $\frac{\sqrt{2}}{2}$ apart.

III If 7 points are placed in the hexagon, two of them are at most 1 apart.

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III
- 14** For a positive integer n , the number $n! = 1 \times 2 \times \cdots \times n$ ends in some number of zeros (for example $10! = 3628800$ ends in two zeros). For how many integers k with $1 \leq k \leq 30$ is there a positive integer n such that $n!$ ends in exactly k zeros?
- A** 20
- B** 23
- C** 24
- D** 25
- E** 29
- F** 30

- 15** A 5×5 board of unit squares, as shown, has one square removed. Someone then tries to cover the remaining 24 squares exactly, with no overlaps and nothing sticking out, using twelve 2×1 dominoes (each covers two squares that share an edge). For how many of the 25 squares is it possible, if that square is the one removed?



- A** 1
- B** 9
- C** 12
- D** 13
- E** 24
- F** 25
- 16** After a meet-up, the guests compare how many hands they shook. Each pair of people shakes hands at most once, and nobody shakes their own hand. Which of the following situations could actually have happened?
- I 9 people, each of whom shook hands with exactly 3 others.
 - II 7 people, each of whom shook hands with exactly 4 others.
 - III 6 people, who shook hands 5, 5, 4, 3, 2, 1 times respectively.
- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

- 17** Some different integers are chosen from $\{1, 2, 3, \dots, 20\}$. Which of the following statements are true?
- I Any 10 chosen integers must include two that have no common factor greater than 1.
 - II Any 11 chosen integers must include two that differ by exactly 1.
 - III Any 11 chosen integers must include two such that one is a multiple of the other.
- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III
- 18** Consider the graph of $y = |x^2 - 4x + 3|$. For $m > 0$, and the equation $|x^2 - 4x + 3| = m|x|$, that is, where the graph meets the V-shaped graph $y = m|x|$. What is the sum of all values of m for which this equation has exactly three distinct real solutions?
- A** $4 - 2\sqrt{3}$
- B** $2\sqrt{3}$
- C** 4
- D** $4 + 2\sqrt{3}$
- E** 8
- F** $8 + 4\sqrt{3}$

19 Let x and y be real numbers. Which of the following statements are true?

- I If x and y are rational, and $x + y$ and xy are integers, then x and y are integers.
- II If $x + y$ and xy are integers, then x and y are integers.
- III If $x + y$ and xy are both odd integers, then x and y are irrational.

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

20 A recording lasts 2025 seconds. It is to be cut into two or more segments whose lengths, in whole seconds, are consecutive positive integers, such as $5, 6, 7, \dots$ in order, with the lengths adding up to 2025 exactly. In how many different ways can this be done? (Two ways are different if their lists of lengths are different.)

- A** 14
- B** 15
- C** 28
- D** 29
- E** 30
- F** 60

Answer grid

1 ____ 2 ____ 3 ____ 4 ____ 5 ____ 6 ____ 7 ____ 8 ____ 9 ____
10 ____ 11 ____ 12 ____ 13 ____ 14 ____ 15 ____ 16 ____ 17 ____ 18 ____
19 ____ 20 ____