

TMUA

Paper 1

Free Mock Paper Solutions

Answer key, worked solutions and score guide.

Questions	20
Time allowed	75 minutes
Calculator	Not allowed
Marking	1 mark per question, no penalty

Answer key

1 D 2 B 3 E 4 E 5 A 6 C 7 E 8 B 9 C

10 D 11 F 12 D 13 C 14 C 15 C 16 B 17 E 18 B

19 D 20 D

Specification codes tested: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8.

Score guide

This table is a rough planning aid. It is **not** an official score: UAT-UK does not publish a conversion for our papers, and real conversions change with every sitting.

Score	Percentage	Suggested next step
0–6	0–30%	Go back to the free lessons for this module, then try the topic sets.
7–10	35–50%	Fix the topics you missed with topic sets, then retake a different mock.
11–14	55–70%	Solid base. Work on speed and the questions you guessed.
15–17	75–85%	Strong. Practise full-length papers back to back.
18–20	90–100%	Excellent. Check for careless slips, then keep timing sharp.

Worked solutions

1. Answer D

1.1

Question: What is the value of $\frac{10}{3 - \sqrt{5}} - \frac{10}{3 + \sqrt{5}}$?

Put the two fractions over a common denominator, $(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$: $\frac{10(3 + \sqrt{5}) - 10(3 - \sqrt{5})}{4} = \frac{20\sqrt{5}}{4} = 5\sqrt{5}$.

Trap. $10\sqrt{5}$ divides by 2 rather than 4 (taking the product of the denominators to be $3^2 - \sqrt{5} = \dots$ or halving only once); 5 mistakes $(\sqrt{5})^2$ for the whole answer.

2. Answer B

1.2

Question: The fourth term of an arithmetic sequence is 17 and the ninth term is 42. What is the sum of the first 20 terms?

Let the first term be a and the common difference d . Then $a + 3d = 17$ and $a + 8d = 42$. Subtracting, $5d = 25$, so $d = 5$ and $a = 2$. The sum of the first 20 terms is $\frac{20}{2}(2a + 19d) = 10(4 + 95) = 990$.

Trap. 970 takes the last term to be $a + 19d = 97$ but then adds only 95 (the $19d$) instead of $2 + 97$; 1980 forgets the factor $\frac{1}{2}$ in $\frac{n}{2}(\dots)$.

3. Answer E

1.1

Question: The mean score of 12 students in a test is 63. The mean score of the 5 girls among them is 70. What is the mean score of the 7 boys?

The total score of all 12 students is $12 \times 63 = 756$ and the girls' total is $5 \times 70 = 350$. So the boys' total is $756 - 350 = 406$ and their mean is $\frac{406}{7} = 58$.

Trap. 56 is $2 \times 63 - 70$, which treats the two groups as if they had equal sizes; the means must be weighted by the numbers (7 boys, 5 girls).

4. Answer E

1.1

Question: The polynomial $p(x) = x^3 + ax^2 + bx - 6$ has $(x - 1)$ as a factor, and leaves remainder 12 when divided by $(x + 1)$. What is the value of a ?

By the factor theorem $p(1) = 0$: $1 + a + b - 6 = 0$, so $a + b = 5$. By the remainder theorem $p(-1) = 12$: $-1 + a - b - 6 = 12$, so $a - b = 19$. Adding, $2a = 24$ and $a = 12$ (and $b = -7$).

Trap. -7 is the value of b , not a ; 19 is $a - b$, found but not finished.

5. Answer A

1.8

Question: The graph of $y = f(x)$ has a maximum point at $(4, -1)$ and no other turning point. The graph of $y = 3 - 2f(x + 1)$ has a turning point. Which of the following describes it?

Build the graph in stages. $y = f(x + 1)$ translates the graph 1 unit to the left: maximum at $(3, -1)$. Multiplying by -2 stretches by factor 2 in the y -direction and reflects in the x -axis, so the maximum becomes a minimum, at $(3, 2)$. Finally adding 3 moves it up: minimum at $(3, 5)$. (Directly: $3 - 2f(x + 1)$ is least when $f(x + 1)$ is greatest, i.e. $x + 1 = 4$, and then equals $3 - 2(-1) = 5$.)

Trap. A minimum at $(5, 5)$ translates in the wrong direction ($x + 1$ moves the graph left, not right); a minimum at $(3, 2)$ forgets the $+3$.

6. Answer C

1.4

Question: A triangle has sides of length 5, 6 and 7. What is its area?

Let θ be the angle between the sides 5 and 6, opposite the side 7. By the cosine rule $7^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cos \theta$, so $49 = 61 - 60 \cos \theta$ and $\cos \theta = \frac{1}{5}$. Then $\sin \theta = \sqrt{1 - \frac{1}{25}} = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$. The area is $\frac{1}{2} \cdot 5 \cdot 6 \sin \theta = 15 \cdot \frac{2\sqrt{6}}{5} = 6\sqrt{6}$.

Trap. 3 uses $\cos \theta = \frac{1}{5}$ as if it were $\sin \theta$ in $\frac{1}{2}ab \sin \theta$; 15 is $\frac{1}{2} \cdot 5 \cdot 6$, with $\sin \theta$ taken as 1.

7. Answer E

1.3

Question: A circle has centre $(2, -3)$ and passes through the point $P(5, 1)$. The tangent to the circle at P crosses the x -axis at the point Q . What is the x -coordinate of Q ?

The radius from the centre $(2, -3)$ to $P(5, 1)$ has gradient $\frac{1 - (-3)}{5 - 2} = \frac{4}{3}$. The tangent is perpendicular to the radius, so its gradient is $-\frac{3}{4}$. Its equation is $y - 1 = -\frac{3}{4}(x - 5)$. Setting $y = 0$: $-1 = -\frac{3}{4}(x - 5)$, so $x - 5 = \frac{4}{3}$ and $x = \frac{19}{3}$.

Trap. $\frac{17}{4}$ uses the gradient $\frac{4}{3}$ of the radius itself as the gradient of the tangent; $\frac{23}{4}$ uses $-\frac{4}{3}$ (the negative, not the negative reciprocal).

8. Answer B

1.7

Question: The trapezium rule with 4 strips of equal width is used to estimate $\int_0^4 \sqrt{x} dx$. What is the estimate?

The strips have width $h = 1$, with ordinates $\sqrt{0} = 0$, $\sqrt{1} = 1$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4} = 2$. The rule gives $\frac{h}{2} [y_0 + y_4 + 2(y_1 + y_2 + y_3)] = \frac{1}{2} [0 + 2 + 2(1 + \sqrt{2} + \sqrt{3})] = 1 + (1 + \sqrt{2} + \sqrt{3}) = 2 + \sqrt{2} + \sqrt{3}$.

Trap. $1 + \sqrt{2} + \sqrt{3}$ leaves out the final ordinate $\sqrt{4} = 2$; $4 + 2\sqrt{2} + 2\sqrt{3}$ forgets the factor $\frac{h}{2} = \frac{1}{2}$; $\frac{16}{3}$ is the exact value of the integral, not the trapezium rule estimate.

9. Answer C

1.1

Question: A bag contains 5 red, 4 blue and 3 green counters. Two counters are taken from the bag at random, one after the other, without replacement. What is the probability that they are the same colour?

Count pairs of counters. There are $\binom{12}{2} = 66$ equally likely pairs. Same colour: red $\binom{5}{2} = 10$, blue $\binom{4}{2} = 6$, green $\binom{3}{2} = 3$, total 19. So the probability is $\frac{19}{66}$. (Equivalently, $\frac{5}{12} \cdot \frac{4}{11} + \frac{4}{12} \cdot \frac{3}{11} + \frac{3}{12} \cdot \frac{2}{11} = \frac{20+12+6}{132} = \frac{19}{66}$.)

Trap. $\frac{25}{72} = \frac{5^2+4^2+3^2}{12^2}$ is the probability if the first counter is replaced before the second is taken.

10. Answer D

1.6

Question: The function $f(x) = x^3 - 6x^2 + kx + 5$, where k is a constant, is decreasing for exactly those values of x that lie in an interval of length 2. What is the local minimum value of f ?

$f'(x) = 3x^2 - 12x + k$, and f is decreasing where $f'(x) < 0$, between the two roots of f' . If these roots are α and β , then $\alpha + \beta = 4$ and $\beta - \alpha = 2$, so the roots are 1 and 3 and $f'(x) = 3(x - 1)(x - 3)$ (so $k = 9$). Then $f(x) = x^3 - 6x^2 + 9x + 5$ has a local maximum at $x = 1$ and a local minimum at $x = 3$, where $f(3) = 27 - 54 + 27 + 5 = 5$.

Trap. 9 is the local *maximum* value, $f(1) = 1 - 6 + 9 + 5 = 9$, not the minimum.

11. Answer F

1.5

Question: Given that $\log_5 2 = p$ and $\log_5 3 = q$, express $\log_{15} 12$ in terms of p and q .

Change to base 5: $\log_{15} 12 = \frac{\log_5 12}{\log_5 15}$. Now $12 = 2^2 \cdot 3$, so $\log_5 12 = 2\log_5 2 + \log_5 3 = 2p + q$. And $15 = 5 \cdot 3$, so $\log_5 15 = \log_5 5 + \log_5 3 = 1 + q$. Hence $\log_{15} 12 = \frac{2p + q}{1 + q}$.

Trap. $\frac{2p + q}{q}$ takes $\log_5 15$ to be q , forgetting $\log_5 5 = 1$; $\frac{2p + q}{p + q}$ writes 15 as 2×3 (confusing it with 6).

12. Answer D

1.4

Question: How many solutions does the equation $\sin 2x = \cos x$ have in the interval $0 \leq x < 2\pi$?

Use $\sin 2x = 2 \sin x \cos x$ to write $2 \sin x \cos x - \cos x = 0$, i.e. $\cos x (2 \sin x - 1) = 0$. Either $\cos x = 0$, giving $x = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\sin x = \frac{1}{2}$, giving $x = \frac{\pi}{6}, \frac{5\pi}{6}$. That is 4 solutions.

Trap. 2 divides both sides by $\cos x$, losing the solutions $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and keeping only $\sin x = \frac{1}{2}$; 3 finds only one of the two solutions of $\cos x = 0$ or of $\sin x = \frac{1}{2}$.

13. Answer C

1.7

Question: What is the total area of the finite regions enclosed by the curve $y = x^3 - 4x$ and the x -axis?

$x^3 - 4x = x(x - 2)(x + 2)$, so the curve meets the x -axis at $x = -2, 0, 2$. Between -2 and 0 the curve is above the axis: area $\int_{-2}^0 (x^3 - 4x) dx = [\frac{x^4}{4} - 2x^2]_{-2}^0 = 0 - (4 - 8) = 4$. Between 0 and 2 it is below the axis, and by symmetry (the curve is odd) the area is also 4. The total area is 8.

Trap. 0 is $\int_{-2}^2 (x^3 - 4x) dx$, in which the two regions cancel because one lies below the axis; 4 counts only one of the two regions.

14. Answer C

1.2

Question: In the expansion of $(1 + ax)^n$ in ascending powers of x , where n is a positive integer and a is a constant, the coefficient of x is 12 and the coefficient of x^2 is 60. What is the coefficient of x^3 ?

The coefficients are $na = 12$ and $\binom{n}{2}a^2 = \frac{n(n-1)}{2}a^2 = 60$. Squaring the first, $n^2a^2 = 144$, so $a^2 = \frac{144}{n^2}$ and the second becomes $\frac{n(n-1)}{2} \cdot \frac{144}{n^2} = \frac{72(n-1)}{n} = 60$. Hence $72n - 72 = 60n$ and $n = 6$, $a = 2$. The coefficient of x^3 is $\binom{6}{3}a^3 = 20 \times 8 = 160$.

Trap. 20 is $\binom{6}{3}$ alone, forgetting the factor $a^3 = 8$.

15. Answer C

1.6

Question: An open rectangular box (with no lid) has a square base. The total area of its base and four sides is 48 cm^2 . What is the greatest possible volume of the box, in cm^3 ?

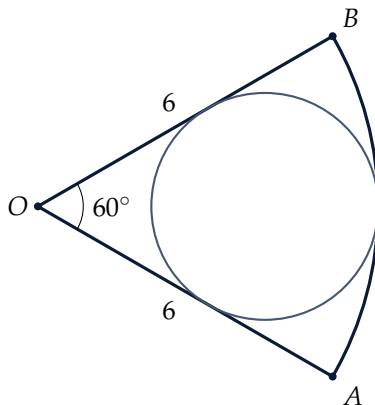
Let the base have side $x \text{ cm}$ and the height be $h \text{ cm}$. The area is $x^2 + 4xh = 48$, so $h = \frac{48 - x^2}{4x}$. The volume is $V = x^2h = \frac{x(48 - x^2)}{4} = 12x - \frac{x^3}{4}$. Then $\frac{dV}{dx} = 12 - \frac{3}{4}x^2 = 0$ gives $x = 4$ (for $x > 0$), and $\frac{d^2V}{dx^2} = -\frac{3}{2}x < 0$, so this is a maximum. Then $V = 12 \cdot 4 - \frac{64}{4} = 48 - 16 = 32$.

Trap. 64 is 4^3 : the volume of a cube with $x = 4$, forgetting that $h = 2$ when $x = 4$; 48 is the area, not the volume.

16. Answer B

1.4

Question: The diagram shows a sector OAB of a circle with centre O and radius 6, in which $\angle AOB = 60^\circ$. A smaller circle lies inside the sector and touches both straight edges OA and OB and the arc AB . What is the area of the region that is inside the sector but outside the smaller circle?



Let the small circle have radius r and centre C . By symmetry C lies on the bisector of angle AOB , which makes 30° with OA . The circle touches OA , so the perpendicular distance from C to OA is r , giving $OC \sin 30^\circ = r$, i.e. $OC = 2r$. It touches the arc internally, so $OC + r = 6$. Hence $3r = 6$ and $r = 2$. The sector has area $\frac{1}{2} \cdot 6^2 \cdot \frac{\pi}{3} = 6\pi$ and the small circle has area $\pi \cdot 2^2 = 4\pi$, so the required area is $6\pi - 4\pi = 2\pi$.

Trap. 4π is the area of the small circle and 6π the area of the sector, each a step on the way rather than the final answer.

17. Answer E

1.5

Question: The positive real numbers a and b satisfy $a^2 + b^2 = 14ab$. What is the value of $\log_2(a + b) - \frac{1}{2} \log_2 a - \frac{1}{2} \log_2 b$?

Add $2ab$ to both sides: $(a + b)^2 = a^2 + b^2 + 2ab = 16ab$. Taking square roots (everything is positive), $a + b = 4\sqrt{ab}$. Now $\log_2(a + b) - \frac{1}{2} \log_2 a - \frac{1}{2} \log_2 b = \log_2 \frac{a + b}{\sqrt{ab}} = \log_2 4 = 2$.

Trap. 4 is the ratio $\frac{a+b}{\sqrt{ab}}$ itself; the question asks for its logarithm to base 2.

18. Answer B

1.2

Question: What is the value of $-1^2 + 2^2 - 3^2 + 4^2 - \dots - 39^2 + 40^2$, that is, of $\sum_{k=1}^{40} (-1)^k k^2$?

Group the terms in pairs: $(2m)^2 - (2m - 1)^2 = (2m - (2m - 1))(2m + (2m - 1)) = 4m - 1$, for $m = 1, 2, \dots, 20$. The sum is the arithmetic series $\sum_{m=1}^{20} (4m - 1) = 4 \cdot \frac{20 \cdot 21}{2} - 20 = 840 - 20 = 820$.

Trap. -820 is the sum with the opposite sign pattern ($1^2 - 2^2 + 3^2 - \dots - 40^2$); 1640 doubles the correct sum.

19. Answer D

1.3

Question: The point P lies on the circle $x^2 + y^2 - 4x - 6y - 12 = 0$. The points O and A are $(0, 0)$ and $(6, 8)$. What is the greatest possible area of triangle OAP ?

Completing the square, $(x - 2)^2 + (y - 3)^2 = 25$: centre $C(2, 3)$, radius 5. Take OA as the base: $OA = \sqrt{36 + 64} = 10$, on the line $4x - 3y = 0$. The area is $\frac{1}{2} \cdot 10 \cdot h = 5h$, where h is the distance from P to the line OA . The distance from C to the line is $\frac{|4 \cdot 2 - 3 \cdot 3|}{5} = \frac{1}{5}$, so h is greatest when P is on the far side of the circle: $h = 5 + \frac{1}{5} = \frac{26}{5}$. The greatest area is $5 \times \frac{26}{5} = 26$.

Trap. 25 takes the greatest height to be just the radius 5, ignoring that the centre is not on the line OA ; 24 uses $5 - \frac{1}{5}$, the nearer side.

20. Answer D

1.2

Question: Ten chairs are placed at equal intervals round a circular table. In how many ways can three of the chairs be chosen so that no two of the chosen chairs are next to each other?

Count the complement. There are $\binom{10}{3} = 120$ ways to choose three chairs. Three consecutive chairs: 10 ways (one for each starting chair). Exactly one adjacent pair (and the third chair not adjacent to the pair): there are 10 adjacent pairs, and the third chair can be any of the $10 - 2 - 2 = 6$ chairs that are not in, or next to, the pair, giving 60 ways. So the number with no two adjacent is $120 - 10 - 60 = 50$.

Trap. $56 = \binom{8}{3}$ is the answer for ten chairs in a *row*; on a circle the first and last chairs are also adjacent.