

TMUA

Paper 2

Free Mock Paper

Mock paper: sit it in one go, under timed conditions.

Questions	20
Time allowed	75 minutes
Calculator	Not allowed
Marking	1 mark per question, no penalty

Instructions

- Answer **all 20 questions**. Each has exactly one correct option.
- You have **75 minutes**. That is about 225 seconds a question, so if you are stuck, move on and return if time allows.
- Each correct answer scores 1 mark. There is no penalty for a wrong or blank answer, so always give an answer.
- You **must not** use a calculator. Use the blank space for rough working.
- Write your answers on the answer grid at the end, or on paper. Mark the paper afterwards with the separate answer key and worked solutions.

Sit this paper in one go, with a timer, in a quiet place. Do not look at the solutions until you have finished. After marking, read the worked solution for every question you missed *and* every question you guessed.

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- 1 The curve $y = x(6 - x)$ and the line $y = 2x$ enclose a finite region. What is its area?
- A $\frac{32}{3}$
- B 12
- C 16
- D $\frac{64}{3}$
- E $\frac{80}{3}$
- F 32
-
- 2 The number $\frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$ can be written as $p + q\sqrt{21}$, where p and q are rational numbers. What is the value of $p + q$?
- A 12
- B 10
- C 7
- D 6
- E 5
- F 3

- 3 What is the sum of all the solutions, in degrees, of the equation $3 \sin x = 2 \cos^2 x$ in the interval $0 \leq x \leq 720$?
- A 1260
- B 1080
- C 900
- D 720
- E 540
- F 180
-
- 4 A geometric series has first term 12 and sum to infinity 16. What is the sum of the first three terms?
- A 12
- B 15
- C $\frac{63}{4}$
- D $\frac{255}{16}$
- E 16
- F $\frac{111}{4}$

- 5 The circle C has equation $x^2 + y^2 - 4x + 6y = 12$, and the point $(6, 0)$ lies on C . The tangent to C at $(6, 0)$ and the two coordinate axes enclose a triangle. What is the area of this triangle?
- A 8
- B 12
- C $\frac{27}{2}$
- D 18
- E 24
- F 32
-
- 6 For each integer k , the curve $y = x^3 + kx^2 + 12x + 1$ is drawn. For how many integer values of k does the curve have no stationary points?
- A 5
- B 9
- C 10
- D 11
- E 12
- F 13

- 7 The positive real numbers x and y satisfy $\log_4 x + \log_2 y = 4$ and $\log_2 x - \log_4 y = 3$. What is the value of $\frac{x}{y}$?
- A 4
- B 8
- C 12
- D 16
- E 20
- F 64
-
- 8 For a positive integer n , let P be the statement “ $n^2 - 1$ is a multiple of 8” and let Q be the statement “ n is odd”. Which of the following is true?
- A Q is a sufficient but not necessary condition for P .
- B Q is a necessary but not sufficient condition for P .
- C Q is a necessary and sufficient condition for P .
- D Q is neither a necessary nor a sufficient condition for P .

- 9 The function f is defined by $f(x) = x^2 - 6x + 5$ for all real x . Which of the following statements are true?
- I The graph of $y = f(x - 2)$ crosses the x -axis at $x = -1$ and $x = 3$.
 - II The graph of $y = f(2x)$ crosses the x -axis at $x = \frac{1}{2}$ and $x = \frac{5}{2}$.
 - III The graph of $y = f(-x)$ crosses the x -axis at $x = -5$ and $x = -1$.
- A None of them
 - B I only
 - C II only
 - D III only
 - E I and II only
 - F I and III only
 - G II and III only
 - H I, II and III

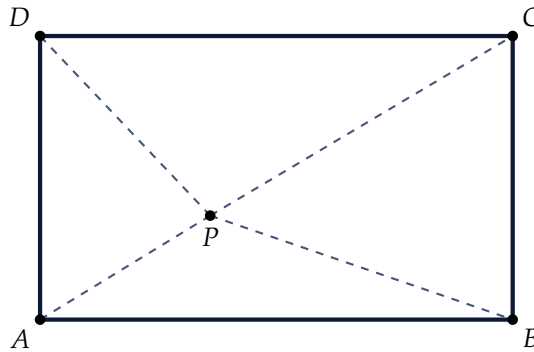
- 10** The diagram shows a rectangle $ABCD$ and a point P inside it. The rectangle and the point shown are not special in any way. Consider the following statements, which refer to any rectangle $ABCD$ and any point P inside it.

I $PA^2 + PC^2 = PB^2 + PD^2$.

II If $PA = PC$, then P lies on the diagonal BD .

III The areas of the triangles PAB and PCD add up to half the area of the rectangle.

Which of these statements are true?



- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

11 A student claims: “if a and b are positive numbers with $a > b$, then $a^b > b^a$ ”. Which of the following pairs (a, b) shows that the claim is false?

A $(a, b) = (2, 1)$

B $(a, b) = (3, 2)$

C $(a, b) = (5, 1)$

D $(a, b) = \left(3, \frac{1}{2}\right)$

E $(a, b) = \left(9, \frac{1}{3}\right)$

F $(a, b) = (4, 3)$

12 A student solves the inequality $\frac{x+1}{x-2} > 3$ for real x as follows.

(1) Multiplying both sides by $x - 2$ gives $x + 1 > 3(x - 2)$.

(2) Expanding, $x + 1 > 3x - 6$.

(3) Rearranging, $7 > 2x$.

(4) Hence $x < \frac{7}{2}$.

(5) The solutions are therefore all real numbers $x < \frac{7}{2}$ with $x \neq 2$.

Which of the following is correct?

A The first error is in line (1).

B The first error is in line (2).

C The first error is in line (3).

D The first error is in line (4).

E The first error is in line (5).

F The argument is correct.

13 In the following statements, m , n and k are integers and x is a real number. Which of the statements are true?

- I For every real number x there is an integer n such that $n > x$.
- II There is an integer n such that $n > x$ for every real number x .
- III For all integers m and n with $m < n$ there is an integer k such that $m < k < n$.

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

14 A two-digit number N has tens digit a and units digit b , where a and b are non-zero. The number N' is formed by reversing the digits of N , so $N = 10a + b$ and $N' = 10b + a$. For example, if $N = 42$ then $N' = 24$. Which of the following must be true?

- I $N + N'$ is a multiple of 11.
- II $N - N'$ is a multiple of 9.
- III $N + N'$ is a multiple of 9.

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

15 For real numbers b and c , let P be the statement “the equation $x^2 + bx + c = 0$ has two distinct positive real roots” and let Q be the statement “ $b < 0$ and $c > 0$ ”. Which of the following is true?

- A** Q is a sufficient but not necessary condition for P .
- B** Q is a necessary but not sufficient condition for P .
- C** Q is a necessary and sufficient condition for P .
- D** Q is neither a necessary nor a sufficient condition for P .

16 Three students each try to prove that $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}$ for all real numbers $a \geq 0$ and $b \geq 0$.

I Both sides are non-negative, and $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab} \geq a + b = (\sqrt{a+b})^2$. Taking positive square roots of both ends gives the result.

II Suppose that $\sqrt{a+b} > \sqrt{a} + \sqrt{b}$. Squaring both sides gives $a + b > a + b + 2\sqrt{ab}$, so $\sqrt{ab} < 0$, which is impossible. So the statement is true.

III If $a = 1$, $b = 1$ then $\sqrt{2} < 2$; if $a = 4$, $b = 9$ then $\sqrt{13} < 5$; and if $a = 0$ the two sides are equal. So the statement holds for all a and b .

Which of these arguments is a valid proof?

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

17 A positive integer is called *odd-handed* if it has an odd number of positive divisors (counting 1 and the number itself). For example, 9 is odd-handed since its divisors are 1, 3 and 9. How many positive integers from 1 to 100 inclusive are odd-handed?

- A** 5
- B** 9
- C** 10
- D** 11
- E** 25
- F** 50

18 The positive real numbers a and b satisfy $a + b = 1$. Which of the following must be true?

I $a^2 + b^2 \geq \frac{1}{2}$.

II $ab \leq \frac{1}{4}$.

III $a^3 + b^3 \geq \frac{1}{4}$.

- A** None of them
- B** I only
- C** II only
- D** III only
- E** I and II only
- F** I and III only
- G** II and III only
- H** I, II and III

- 19** A counter moves along the lines of a grid from $(0,0)$ to $(5,5)$, each move being one unit to the right or one unit up. How many different routes avoid both of the points $(2,2)$ and $(3,3)$?
- A** 252
- B** 168
- C** 132
- D** 84
- E** 36
- F** 12
-
- 20** Each of the nine cells of a 3×3 grid is filled with either 0 or 1. Each of the four 2×2 blocks of neighbouring cells contains exactly two 1s. How many different fillings of the grid are possible?
- A** 6
- B** 8
- C** 12
- D** 14
- E** 16
- F** 18

Answer grid

1 ____ 2 ____ 3 ____ 4 ____ 5 ____ 6 ____ 7 ____ 8 ____ 9 ____
10 ____ 11 ____ 12 ____ 13 ____ 14 ____ 15 ____ 16 ____ 17 ____ 18 ____
19 ____ 20 ____

End of paper.