

TMUA

Paper 1

Coordinate geometry and circles

Free lesson notes

Covers	1.3 straight lines, circles, tangents and normals, distances
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

Almost every TMUA paper has two or three questions that are secretly about circles and lines. They look like algebra problems (“two circles of radius r and 3...”) but the quick route is geometric: the distance between two centres, a right angle in a semicircle, a tangent that is perpendicular to a radius. Students who set up coordinates and grind lose minutes; students who *draw the picture first* often finish in under a minute.

This lesson gives you the working toolkit (lines, distances, areas, the equation of a circle), then the recurring patterns: touching circles, common chords, loci that turn out to be circles, tangents and normals. You will meet ten worked examples. Every one is original, uses numbers you can handle without a calculator, and has been checked by computer as well as by hand.

How to use this lesson. Read a section, then cover each worked example and try it before reading the solution. The TMUA gives you about 90 seconds a question on average, so after every idea ask two things: what is the quickest sound method, and what would a quick sketch or a test value tell me?

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The patterns to expect

What the TMUA keeps asking

- **Two circles, one unknown radius:** given one circle and the centre of another, find the radius so that they touch (usually *two* answers: external and internal tangency), find the difference of those two radii, or the range of radii for which they meet in two points.
- **Shortest or longest distance between two circles,** or from a point to a circle. It always runs along the line of centres.
- **Common chord:** subtract the two circle equations to get a line, then find its length or its intersection with an axis. If the radii are equal the chord is the perpendicular bisector of the centres.
- **A right angle hidden in a locus:** “how many points P on this line make $\angle APB = 90^\circ$?” The points lie on the circle with diameter AB , and you count how many times the line cuts it.
- **Tangent and normal** to a circle (perpendicular to the radius) or to a curve (use the derivative), and the length of a tangent from an outside point.
- **Common tangent of two circles:** the angle it makes with the line of centres, or its length, from one right-angled triangle.
- **Rectangles, triangles and areas** built from points and perpendicular lines, where you must find a fourth vertex or an area from coordinates.

Notice that none of these needs a long computation. In every case there is either a one-line geometric fact or a quadratic whose discriminant answers the question. The first habit to build is to **draw the picture and mark the centres before you write any algebra**.

Lines, distances and areas

You need these cold. For points (x_1, y_1) and (x_2, y_2) :

Key ideas

gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$ midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ distance $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 line through (x_1, y_1) with gradient m : $y - y_1 = m(x - x_1)$. Parallel lines: $m_1 = m_2$. Perpendicular lines: $m_1 m_2 = -1$ (a vertical line is perpendicular to a horizontal one; there are no gradients to multiply).

Area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|.$$

The other way is to draw the triangle inside a rectangle with sides parallel to the axes and subtract the corner triangles, or to use $\frac{1}{2} \times \text{base} \times \text{height}$ when a side is horizontal, vertical, or a known perpendicular bisector.

Strategy: equidistance means perpendicular bisector

The points equidistant from A and B form the *perpendicular bisector* of AB . To use this, write $PA^2 = PB^2$ with the unknown coordinates in P . Squares avoid square roots, and the x^2 and y^2 terms cancel, so you always get a *linear* equation.

Worked example: equidistant point and an area

Question. $A = (1, 5)$ and $B = (7, 1)$. The point P lies on the x -axis and is the same distance from A and from B . Find P and the area of triangle ABP .

Solution. Let $P = (p, 0)$. Then $PA^2 = PB^2$ gives $(p - 1)^2 + 25 = (p - 7)^2 + 1$, so $p^2 - 2p + 26 = p^2 - 14p + 50$, hence $12p = 24$ and $p = 2$. So $P = (2, 0)$.

For the area, P lies on the perpendicular bisector of AB , so $PM \perp AB$ where $M = (4, 3)$ is the midpoint. Then $AB^2 = 6^2 + 4^2 = 52$ and $PM^2 = 2^2 + 3^2 = 13$, so the area is $\frac{1}{2} \sqrt{52} \sqrt{13} = \frac{1}{2} \sqrt{676} = 13$.

Examiner's eye. The shoelace formula gives the same answer: $\frac{1}{2} |1(1 - 0) + 7(0 - 5) + 2(5 - 1)| = \frac{1}{2} |1 - 35 + 8| = 13$. When a question hands you a perpendicular bisector, the base-times-height route is usually faster and has less arithmetic to go wrong.

Worked example: completing a rectangle

Question. $PQRS$ is a rectangle with $P = (3, 1)$ and $Q = (7, 3)$. The vertex S lies on the y -axis. Find R and the area of the rectangle.

Solution. PQ has gradient $\frac{3-1}{7-3} = \frac{1}{2}$, so PS has gradient -2 : $y - 1 = -2(x - 3)$. At $x = 0$, $y = 7$, so $S = (0, 7)$.

In a rectangle $\vec{QR} = \vec{PS} = (-3, 6)$, so $R = (7 - 3, 3 + 6) = (4, 9)$. The sides are $PQ = \sqrt{16 + 4} = 2\sqrt{5}$ and $PS = \sqrt{9 + 36} = 3\sqrt{5}$, so the area is $2\sqrt{5} \times 3\sqrt{5} = 30$.

Examiner's eye. Quick sanity test: the diagonals PR and QS must share a midpoint. PR has midpoint $(3.5, 5)$ and so does QS . A fourth vertex is almost always found by *adding a vector*, not by solving simultaneous equations.

Common mistakes

- Using $m_1 m_2 = -1$ when one line is vertical, or forgetting that a horizontal line has a vertical perpendicular.
- Writing the midpoint with a subtraction, or the distance without the square root. Always test on a simple pair like $(0, 0), (6, 8)$.
- Finding the *equation* of the perpendicular bisector when the question only needs the equidistance condition $PA^2 = PB^2$.

Circles: equation, tangent and line

A circle with centre (a, b) and radius r has equation $(x - a)^2 + (y - b)^2 = r^2$. The TMUA usually gives it expanded, $x^2 + y^2 + px + qy + c = 0$, so you must **complete the square** in both variables:

$$x^2 + px + y^2 + qy = -c \implies \left(x + \frac{p}{2}\right)^2 + \left(y + \frac{q}{2}\right)^2 = \frac{p^2}{4} + \frac{q^2}{4} - c.$$

So the centre is $\left(-\frac{p}{2}, -\frac{q}{2}\right)$ and $r^2 = \frac{p^2 + q^2}{4} - c$, which must be positive for a circle to exist.

Strategy: what "touches an axis" means

A circle touches the x -axis exactly when the *distance from the centre to the axis equals the radius*: $|b| = r$. For the y -axis it is $|a| = r$. Algebraically, putting $y = 0$ must give a quadratic in x with $\Delta = 0$ (a repeated root). Use whichever is quicker; the first is almost always quicker.

Worked example: a circle with a parameter

Question. The equation $x^2 + y^2 - 8x + 6y + k = 0$ describes a circle for some values of k . For which k does it exist? Find k if the circle touches (a) the x -axis, (b) the y -axis.

Solution. Complete the square: $(x - 4)^2 + (y + 3)^2 = 25 - k$. The centre is $(4, -3)$ and $r^2 = 25 - k$, so a circle needs $k < 25$. (a) The centre is 3 from the x -axis, so $r = 3$, $25 - k = 9$ and $k = 16$. (b) The centre is 4 from the y -axis, so $r = 4$, $25 - k = 16$ and $k = 9$.

Examiner's eye. Check (a) by putting $y = 0$: $x^2 - 8x + 16 = (x - 4)^2 = 0$, a repeated root, so the circle meets the axis at the single point $(4, 0)$. Note that the same circle cannot do both: touching one axis does not make it touch the other.

Tangent and normal at a point on a circle. The tangent is *perpendicular to the radius*, and the normal is the line along the radius (so it passes through the centre). The gradient of the radius from the centre C to the point P is m , then the tangent has gradient $-1/m$. If T is a point outside the circle and TP touches the circle at P , then $CP \perp TP$ and Pythagoras gives

$$TP^2 = TC^2 - r^2.$$

Worked example: tangent, normal and tangent length

Question. The circle $x^2 + y^2 - 6x - 2y - 15 = 0$ passes through $P = (7, 4)$. Find the equation of the tangent at P . The tangent meets the x -axis at T : find T and the length TP without using the distance formula between T and P .

Solution. Completing the square: $(x - 3)^2 + (y - 1)^2 = 25$, centre $C = (3, 1)$, radius 5. (Check: $(7 - 3)^2 + (4 - 1)^2 = 25$.)

The radius CP has gradient $\frac{4-1}{7-3} = \frac{3}{4}$, so the tangent has gradient $-\frac{4}{3}$: $y - 4 = -\frac{4}{3}(x - 7)$, i.e. $4x + 3y = 40$. Putting $y = 0$ gives $T = (10, 0)$.

Now $TC^2 = 7^2 + 1^2 = 50$, so $TP^2 = TC^2 - r^2 = 50 - 25 = 25$ and $TP = 5$.

Examiner's eye. As a check, $TP = \sqrt{3^2 + 4^2} = 5$ directly. Also check the tangent line is at distance 5 from the centre:

$$\frac{|4(3) + 3(1) - 40|}{\sqrt{4^2 + 3^2}} = \frac{25}{5} = 5.$$

Does a line meet a circle? There are two equally good methods. (1) Substitute the line into the circle and look at the discriminant of the resulting quadratic: $\Delta > 0$ two points, $\Delta = 0$ tangent, $\Delta < 0$ none. (2) Compare the *distance* δ from the centre to the line with the radius: $\delta < r$ two points, $\delta = r$ tangent, $\delta > r$ none. The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Strategy: compare distance with radius

For a line and a circle with a parameter in the line, method (2) is shorter: one fraction against r . Method (1) is safer if you cannot remember the distance formula. Do the one you trust, and use the other as a check if time allows.

Worked example: which lines meet the circle?

Question. For which values of k does the line $y = 2x + k$ meet the circle $(x - 1)^2 + (y + 2)^2 = 5$ (at one or two points)?

Solution. By distance. The line is $2x - y + k = 0$, the centre is $(1, -2)$ and the radius is $\sqrt{5}$. The distance from the centre to the line is $\frac{|2 + 2 + k|}{\sqrt{5}}$, so the line meets the circle when $\frac{|k + 4|}{\sqrt{5}} \leq \sqrt{5}$, that is $|k + 4| \leq 5$, so $-9 \leq k \leq 1$.

By discriminant. Substituting $y = 2x + k$: $5x^2 + (4k + 6)x + (k + 2)^2 - 4 = 0$, and $\Delta = (4k + 6)^2 - 20(k^2 + 4k) = -4k^2 - 32k + 36 = -4(k + 9)(k - 1)$. This is ≥ 0 for $-9 \leq k \leq 1$. Same answer.

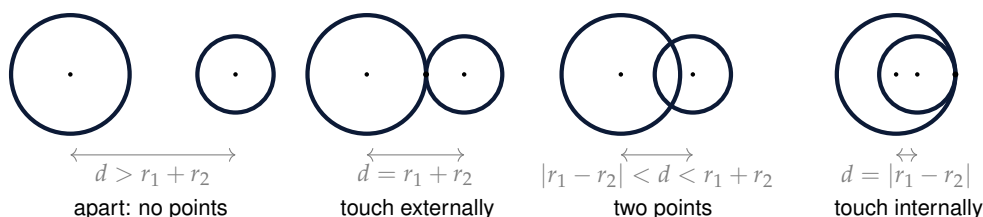
Examiner's eye. The two end values $k = -9$ and $k = 1$ are the *tangent* lines, and "meet" includes tangency, so the inequality is non-strict. If the question had said "cuts the circle at two points" the answer would be $-9 < k < 1$.

Common mistakes

- Forgetting the sign when reading the centre: $(y + 3)^2$ means $b = -3$, not 3.
- Dividing the whole equation by 2 when the circle is written $2x^2 + 2y^2 + \dots = 0$. Divide first so that the x^2 and y^2 coefficients are 1, otherwise r^2 comes out wrong.
- Using the gradient of the radius as the gradient of the tangent. The tangent's gradient is the *negative reciprocal*.

Two circles

The whole story of two circles depends on one number: d , the distance between their centres, compared with the radii $r_1 \geq r_2$.



Read the picture from left to right: as d shrinks, the circles go from apart, to touching, to crossing, to one inside the other touching at a single point (and then, for even smaller d , one circle lies completely inside the other with no common points). The shortest distance between two circles that are apart is $d - r_1 - r_2$, measured along the line of centres.

Strategy: compute d first, then compare

Whenever a question mentions two circles, find d immediately. Then “touch” means $d = r_1 + r_2$ or $d = |r_1 - r_2|$, so a missing radius usually has **two** answers, and “meet at two points” is the open interval $|r_1 - r_2| < d < r_1 + r_2$.

Worked example: an unknown radius

Question. C_1 is the circle $x^2 + y^2 = 9$. C_2 has centre $(8, 6)$ and radius r . (a) Find the two values of r for which the circles touch, and the difference between them. (b) For which r do the circles meet at exactly two points?

Solution. C_1 has centre O and radius 3; the distance between the centres is $d = \sqrt{8^2 + 6^2} = 10$. (a) External contact: $r + 3 = 10$, so $r = 7$. Internal contact (the big circle swallows the small one): $r - 3 = 10$, so $r = 13$. The difference is $13 - 7 = 6$. (b) Two points need $|r - 3| < 10 < r + 3$. The right inequality gives $r > 7$; the left gives $r - 3 < 10$, i.e. $r < 13$ (and $3 - r < 10$ is automatic). So $7 < r < 13$.

Examiner's eye. A neat way to see the answer: the points of C_1 lie at distances from $(8, 6)$ ranging from $10 - 3 = 7$ to $10 + 3 = 13$. The circle C_2 meets C_1 in two points exactly when r is strictly between the nearest and farthest distances, and touches when r equals one of them. The difference between the two touching radii is therefore the diameter $2 \times 3 = 6$ of C_1 .

The common chord. When two circles cross, the line through their two intersection points is the *common chord*. Its equation comes free: if the circles are $S_1 = 0$ and $S_2 = 0$ (both written with $x^2 + y^2$ coefficient 1), then $S_1 - S_2 = 0$ is a line, because the x^2 and y^2 terms cancel, and every common point satisfies it. It is perpendicular to the line of centres.

Strategy: subtract, then substitute back

Subtract the equations to get the line of the chord; substitute it into *either* circle to find the two end points. If the circles have **equal radius**, the chord is the perpendicular bisector of the line joining the centres, and its length is $2\sqrt{r^2 - (d/2)^2}$.

Worked example: the common chord

Question. Two circles are $x^2 + y^2 = 20$ and $(x - 6)^2 + (y - 2)^2 = 20$. Find the equation of the line through their points of intersection and the length of the common chord.

Solution. Expanding the second: $x^2 + y^2 - 12x - 4y + 40 - 20 = 0$, i.e. $x^2 + y^2 - 12x - 4y + 20 = 0$. Subtracting from $x^2 + y^2 - 20 = 0$ gives $12x + 4y - 40 = 0$, so $3x + y = 10$. Put $y = 10 - 3x$ into $x^2 + y^2 = 20$: $10x^2 - 60x + 80 = 0$, so $x^2 - 6x + 8 = 0$, $x = 2$ or 4 . The points are $(2, 4)$ and $(4, -2)$, and the chord has length $\sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$.

Examiner's eye. Geometric check using equal radii: $d = \sqrt{36 + 4} = \sqrt{40}$, so half the chord is $\sqrt{20 - 10} = \sqrt{10}$ and the chord is $2\sqrt{10}$. The chord passes through the midpoint $(3, 1)$ of the centres, and $3(3) + 1 = 10$ confirms it.

Common tangents. A line that touches both circles on the same side (an *external* common tangent) can be found from a right-angled triangle. Join the two centres, draw the radii to the points of contact (both are perpendicular to the tangent), and drop a perpendicular from the centre of the smaller circle onto the larger radius.

Worked example: angle of a common tangent

Question. Circle A has centre $(0,0)$ and radius 3; circle B has centre $(4,0)$ and radius 1, so they touch externally. A common tangent touches A at P and B at Q , on the same side of the line AB . Find the length PQ and the acute angle between PQ and AB .

Solution. Draw the radii AP and BQ ; both are perpendicular to PQ , so they are parallel. From B draw the perpendicular BX to AP . Then $PXBQ$ is a rectangle, so $PQ = XB$ and $AX = AP - XP = 3 - 1 = 2$.

In the right-angled triangle AXB the hypotenuse is $AB = 4$ and $AX = 2$, so $\cos \angle XAB = \frac{1}{2}$, giving $\angle XAB = 60^\circ$, and $XB = \sqrt{16 - 4} = 2\sqrt{3}$. So $PQ = 2\sqrt{3}$.

The tangent is perpendicular to AP , which makes 60° with AB , so the tangent makes $90^\circ - 60^\circ = 30^\circ$ with AB .

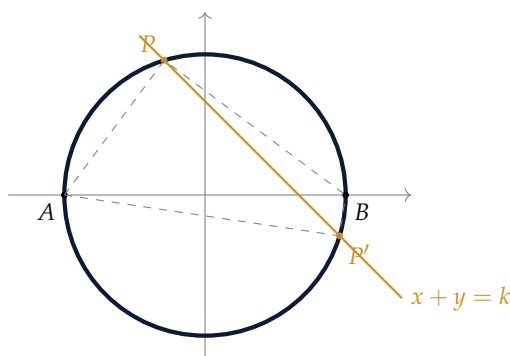
Examiner's eye. General rule: the external common tangent has length $\sqrt{d^2 - (r_1 - r_2)^2}$, and $\sin \theta = \frac{r_1 - r_2}{d}$ for the angle θ it makes with the line of centres. Here $\sin \theta = \frac{2}{4} = \frac{1}{2}$, so $\theta = 30^\circ$. Both routes agree.

Common mistakes

- Giving only the external-tangency radius. Whenever the question says "touch", test internal tangency too.
- Subtracting circle equations that are not both in the form $x^2 + y^2 + \dots = 0$ (for instance, one still has $2x^2 + 2y^2$). Normalise first.
- Thinking "circles meet" needs $d < r_1 + r_2$ only. You also need $d > |r_1 - r_2|$, otherwise one circle sits inside the other.

Right angles, semicircles and loci

The angle in a semicircle is 90° , and conversely: **if $\angle APB = 90^\circ$ then P lies on the circle with diameter AB** . So the locus (the set of all possible positions) of a point P that sees a segment AB at a right angle is a circle: centre at the midpoint of AB , radius half of AB . The only exceptions are A and B themselves, where no angle is defined.



the circle on diameter AB : every point P on it gives $\angle APB = 90^\circ$

Strategy: turn the right angle into a circle

When a question asks how many points make $\angle APB = 90^\circ$ (or for the position of such a point on a line, or an area), **write down the circle on AB as diameter** and then it is a line-circle problem: compare the distance from the centre to the line with the radius. Remember to *remove A and B* if the line goes through either of them. The algebraic alternative is $\vec{PA} \cdot \vec{PB} = 0$, or $m_{PA}m_{PB} = -1$, which gives the same circle.

Worked example: counting right-angled points

Question. $A = (-3,0)$ and $B = (3,0)$. For each value of the constant k , how many points P on the line $x + y = k$ satisfy $\angle APB = 90^\circ$?

Solution. The points with $\angle APB = 90^\circ$ lie on the circle with diameter AB (except A and B): $x^2 + y^2 = 9$, centre O , radius 3. The line $x + y - k = 0$ is at distance $\frac{|k|}{\sqrt{2}}$ from O . Tangent when $\frac{|k|}{\sqrt{2}} = 3$, i.e. $|k| = 3\sqrt{2} \approx 4.24$. So the line cuts the circle twice for $|k| < 3\sqrt{2}$ and misses it for $|k| > 3\sqrt{2}$.

Now remove A and B . The line passes through $B = (3,0)$ when $k = 3$, and through $A = (-3,0)$ when $k = -3$. In those cases one of the two intersections is not allowed. Therefore:

- $|k| < 3\sqrt{2}$ and $k \neq \pm 3$: **two** points;
- $k = \pm 3$ or $|k| = 3\sqrt{2}$: **one** point;
- $|k| > 3\sqrt{2}$: **none**.

Examiner's eye. Test $k = 3$ directly: $y = 3 - x$ and $x^2 + y^2 = 9$ give $2x^2 - 6x = 0$, so $x = 0$ or $x = 3$. The root $x = 3$ is B itself, leaving $P = (0, 3)$, and indeed $\vec{PA} = (-3, -3)$ and $\vec{PB} = (3, -3)$ have dot product $-9 + 9 = 0$.

Normals and tangents to curves

Circles are not the only curves with tangents and normals. For any curve $y = f(x)$, the **tangent** at $x = a$ has gradient $f'(a)$, and the **normal** has gradient $-1/f'(a)$ (when $f'(a) \neq 0$; if $f'(a) = 0$ the normal is the vertical line $x = a$). Normals are very common on the TMUA because the gradient algebra is a little fiddly.

Strategy: a known root gives the other for free

When a normal or tangent at $x = a$ meets the curve *again*, substituting the line into the curve gives a polynomial that **already has $x = a$ as a root** (a double root for a tangent). Don't expand fully and factorise by guessing: use the sum or the product of roots. For a quadratic $px^2 + qx + s = 0$ the roots add to $-q/p$.

Worked example: where the normal meets the curve again

Question. The normal to $y = x^2$ at $P = (1, 1)$ meets the curve again at Q . Find Q and the length PQ .

Solution. $y' = 2x$, so at $x = 1$ the tangent has gradient 2 and the normal has gradient $-\frac{1}{2}$: $y - 1 = -\frac{1}{2}(x - 1)$. Setting $x^2 = 1 - \frac{1}{2}(x - 1)$ gives $2x^2 + x - 3 = 0$. The roots add up to $-\frac{1}{2}$, and one root is 1, so the other is $-\frac{3}{2}$. (Factorising: $(2x + 3)(x - 1) = 0$.) Then $Q = (-\frac{3}{2}, \frac{9}{4})$.

$PQ^2 = (\frac{5}{2})^2 + (\frac{5}{4})^2 = \frac{25}{4} + \frac{25}{16} = \frac{125}{16}$, so $PQ = \frac{5\sqrt{5}}{4}$.

Examiner's eye. Notice that PQ is a multiple of the vector $(2, 1)$: the line has direction $(2, -1)$ scaled. Because the normal has gradient $-\frac{1}{2}$, every displacement along it is a multiple of $(2, -1)$, which is a good way to avoid slips in the final distance.

Common mistakes

- Using the tangent gradient for the normal, or flipping the reciprocal but forgetting the minus sign.
- Expanding to a quartic or a cubic when the known root lets you reduce the degree by one at once.

Habits that win marks

- **Draw, then compute.** Mark centres, radii and the line of centres. Most circle questions are one right-angled triangle.
- **Normalise equations.** Make the x^2 and y^2 coefficients equal to 1 before completing the square or subtracting.
- **Two answers.** "Touch" means external *or* internal. A square root gives $\pm$. Say which cases you have excluded and why.
- **Check the end points.** Is the question about meeting (non-strict) or crossing (strict)? Are A and B excluded?
- **Verify one value.** Put your final answer back into the original picture: is the point on the circle, is the tangent at the right distance from the centre?
- **Spend 2 minutes, then move on.** If the algebra turns into a long expansion you have missed the geometric shortcut. Mark the question and return.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Find the centre and radius of $x^2 + y^2 + 10x - 4y + 13 = 0$.
2. Find the equation of the perpendicular bisector of the points $(1, 3)$ and $(5, -1)$.
3. A circle with centre $(0, 0)$ and radius 2 touches externally a circle with centre $(3, 4)$. Find the radius of the second circle.
4. Find the shortest distance between the circles $x^2 + y^2 = 4$ and $(x - 6)^2 + (y - 8)^2 = 9$.
5. Find the area of the triangle with vertices $(0, 0)$, $(6, 2)$ and $(2, 6)$.
6. For which values of c is the line $y = x + c$ tangent to $x^2 + y^2 = 18$?
7. Find the tangent to $x^2 + y^2 = 13$ at the point $(2, 3)$.
8. $A = (0, 0)$ and $B = (0, 4)$. How many points P on the line $x = 2$ have $\angle APB = 90^\circ$?

Answers to the self-check

1. $(x+5)^2 + (y-2)^2 = 25 + 4 - 13 = 16$: centre $(-5, 2)$, radius 4.
2. Midpoint $(3, 1)$; gradient of the segment $\frac{-1-3}{5-1} = -1$, so the bisector has gradient 1: $y - 1 = x - 3$, i.e. $y = x - 2$.
3. $d = 5$ and $d = 2 + r$, so $r = 3$.
4. $d = 10$; shortest distance $= 10 - 2 - 3 = 5$.
5. $\frac{1}{2}|6 \cdot 6 - 2 \cdot 2| = \frac{1}{2}(36 - 4) = 16$.
6. The distance from the origin is $\frac{|c|}{\sqrt{2}} = 3\sqrt{2}$, so $|c| = 6$ and $c = \pm 6$.
7. The radius has gradient $\frac{3}{2}$, so the tangent has gradient $-\frac{2}{3}$: $y - 3 = -\frac{2}{3}(x - 2)$, i.e. $2x + 3y = 13$.
8. They lie on the circle with centre $(0, 2)$ and radius 2. The line $x = 2$ is at distance 2 from the centre, so it is a tangent: exactly 1 point, $(2, 2)$.

Next step

Now do the matching topic practice set: 20 questions in TMUA style, with worked solutions. Mark it, then redo the ones you missed a week later without looking at the solutions.