

TMUA

Paper 1

Differentiation

Free lesson notes

Covers	1.6 differentiation, stationary points, tangents and normals, optimisation
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

Differentiation on the TMUA is never a drill. You will not be asked to differentiate a long, ugly function for its own sake. Instead the derivative is a *tool*: it finds the turning points that decide how many roots a cubic has, it gives the gradient of a normal, it locates a minimum that you then plug into something else, and it tells you where a function is going up or down. Questions often arrive disguised: a quotient with a surd in the denominator, a stationary point whose position depends on a parameter, or a graph of f' and a question about f .

This lesson teaches the handful of ideas that cover almost every differentiation question in the real papers: rewriting before you differentiate, reading stationary points and the sign of f' , tangents and normals, and optimisation. There are thirteen worked examples, each tied to a pattern that recurs, and each was checked by computer as well as by hand. The most valuable habit it tries to build is this: *before you differentiate anything, sketch the shape the question is describing*.

How to use this lesson. Read a section, then cover each worked example and try it before reading the solution. The TMUA gives you about 90 seconds a question on average, so after every idea ask two things: what is the quickest sound method, and what would a quick sketch or a test value tell me?

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The patterns to expect

What the TMUA keeps asking

- **Differentiate after rewriting:** a quotient such as $\frac{x^4 - 7x}{3x\sqrt{x}}$ that must be split into separate powers of x (including fractional and negative powers) before you can differentiate it, often followed by substituting a value.
- **Stationary points with a parameter:** a function whose stationary point depends on b , followed by a second question about that point, for example the least possible distance from the origin to it. You minimise the *square* of the distance as a function of b .
- **Three real roots of a cubic:** a parameter k or p in a cubic, and the question “for which values does $f(x) = 0$ have three distinct real roots?” The answer comes from the two stationary values, not from solving the cubic.
- **Tangent and normal:** “the line $y = mx + c$ is the normal to the curve at (p, q) ” with unknowns to find, or a tangent through a point that is *not* on the curve.
- **Reading the graph of f' :** a sketch of the derivative is given and you must decide where f has a local minimum, a local maximum or neither.
- **Increasing and decreasing:** on what fraction of an interval is f decreasing? Or a “which statements must be true” question where $f' > 0$ everywhere is *not* the same as $f > 0$ everywhere.
- **Optimisation and end-points:** a quantity that depends on time or one length, and a request for its greatest or least value on a stated interval.

The routine skills (power rule, tangent, stationary point) must be automatic. Every minute you save on them is a minute for the thinking that the real marks reward.

Rewrite first, then differentiate

The only rule you need for most TMUA differentiation is

$$\frac{d}{dx} x^n = nx^{n-1},$$

and it holds for *every* rational n : positive, negative or fractional. The skill is therefore not the differentiating but the **rewriting**. Convert everything to powers of x :

$$\sqrt{x} = x^{1/2}, \quad \frac{1}{x} = x^{-1}, \quad \frac{1}{\sqrt{x}} = x^{-1/2}, \quad x\sqrt{x} = x^{3/2}, \quad \sqrt[3]{x^2} = x^{2/3}.$$

A fraction with a single term underneath splits into one term per numerator term: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$. After that each term is a constant times a power of x , and you differentiate term by term. There is no need for the quotient rule, and it would only slow you down.

Strategy: Split, convert, differentiate

(1) Divide each numerator term by the denominator, using index laws: $\frac{x^m}{x^n} = x^{m-n}$. (2) Write every term as $c x^{\text{power}}$. (3) Differentiate term by term: multiply by the power, then subtract one from it. (4) Only then substitute numbers, converting back with $x^{3/2} = (\sqrt{x})^3$, which is easy for $x = 4, 9, 16 \dots$

Worked example: a quotient with a surd

Question. Let $y = \frac{x^4 - 7x}{3x\sqrt{x}}$ for $x > 0$. Find $\frac{dy}{dx}$ when $x = 4$.

Solution. The denominator is $3x^{3/2}$, so

$$y = \frac{1}{3} (x^{5/2} - 7x^{-1/2}) \quad (\text{since } \frac{x^4}{x^{3/2}} = x^{5/2}, \frac{x}{x^{3/2}} = x^{-1/2}).$$

Differentiating: $\frac{dy}{dx} = \frac{1}{3} \left(\frac{5}{2}x^{3/2} + \frac{7}{2}x^{-3/2} \right)$. At $x = 4$ we have $x^{3/2} = 8$ and $x^{-3/2} = \frac{1}{8}$, so

$$\frac{dy}{dx} = \frac{1}{3} \left(20 + \frac{7}{16} \right) = \frac{1}{3} \cdot \frac{327}{16} = \frac{109}{16}.$$

Examiner's eye. Notice the sign: $-7x^{-1/2}$ differentiates to $+\frac{7}{2}x^{-3/2}$ (two negatives). A slip here is the most common loss of marks. Also notice that you chose to substitute $x = 4$ *after* differentiating, never before.

The derivative from first principles. The gradient at x is the limit of the gradient of a chord whose width h shrinks to zero:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

For $f(x) = x^2$ the chord gradient is $\frac{(x+h)^2 - x^2}{h} = 2x + h$, which tends to $2x$. You will rarely be asked to do this from scratch, but you may be shown a limit and asked for its value. The trick is to *recognise it as a derivative*.

Worked example: a limit that is secretly a derivative

Question. Evaluate (a) $\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h}$ and (b) $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$.

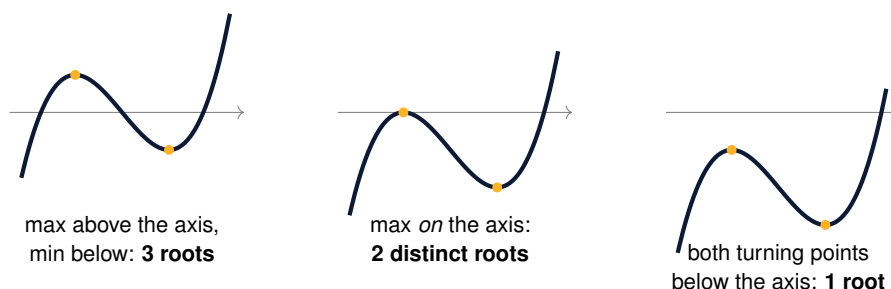
Solution. (a) Since $32 = 2^5$ this is $\frac{f(2+h) - f(2)}{h}$ for $f(x) = x^5$, so it equals $f'(2) = 5 \cdot 2^4 = 80$. (b) Since $2 = \sqrt{4}$ this is the derivative of $f(x) = \sqrt{x}$ at $x = 4$: $f'(x) = \frac{1}{2}x^{-1/2}$, so the limit is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

Examiner's eye. Never expand $(2+h)^5$. Spot the pattern $f(a+h) - f(a)$ over h , identify f and a , and differentiate.

Stationary points, and when a cubic has three roots

A **stationary point** of $y = f(x)$ is where $f'(x) = 0$. It is a **local maximum** if f' changes from $+$ to $-$ as x increases, a **local minimum** if it changes from $-$ to $+$, and neither (a **stationary point of inflection**) if f' does not change sign. The second derivative shortcut ($f'' > 0$ minimum, $f'' < 0$ maximum) is quick, but when $f'' = 0$ it tells you nothing, and you must fall back on the sign of f' either side.

Counting the roots of a cubic. A cubic with positive leading coefficient climbs from $-\infty$, may turn down at a local maximum, turn up at a local minimum, and then climbs to $+\infty$. The number of times it crosses the x -axis depends only on where those two turning points sit relative to the axis:



So **three distinct real roots** occur exactly when the local maximum is above the axis *and* the local minimum is below it, that is $f(\text{max}) > 0 > f(\text{min})$. If either stationary value is exactly 0 you get a repeated root and only two distinct roots. If both stationary values have the same sign you get one root. (A cubic with a negative leading coefficient is the same picture turned over, and the same condition on the two stationary values works.)

Strategy: stationary values decide the roots

For "three distinct real roots" do not try to solve the cubic. (1) Differentiate and find the two stationary x -values. (2) Evaluate f at both; these are expressions in the parameter. (3) Require $\text{max} > 0$ and $\text{min} < 0$. (4) Combine the two inequalities, and check the end-points separately (strict inequalities, because equality gives a repeated root).

Worked example: a range for k

Question. Find all values of k for which $2x^3 - 9x^2 + 12x - k = 0$ has three distinct real roots.

Solution. Let $f(x) = 2x^3 - 9x^2 + 12x - k$. Then $f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2)$, so the stationary points are at $x = 1$ (the local maximum, since the leading coefficient is positive) and $x = 2$ (the local minimum). The stationary values are

$$f(1) = 2 - 9 + 12 - k = 5 - k, \quad f(2) = 16 - 36 + 24 - k = 4 - k.$$

Three distinct roots need $5 - k > 0$ and $4 - k < 0$, so $4 < k < 5$.

Examiner's eye. Equivalently: the horizontal line $y = k$ must cut the curve $y = 2x^3 - 9x^2 + 12x$ three times. That curve has a local maximum 5 and a local minimum 4, so k must lie strictly between them. Thinking of "a line at height k crossing a fixed cubic" is often easier than moving the cubic.

Worked example: a parameter in the turning points

Question. Find the set of values of p for which $x^3 - 3px + 2 = 0$ has three distinct real roots.

Solution. $f'(x) = 3x^2 - 3p$. If $p \leq 0$ then $f' \geq 0$ and the cubic is increasing, so there is only one root. Take $p > 0$: the stationary points are at $x = \pm\sqrt{p}$, the maximum at $x = -\sqrt{p}$ and the minimum at $x = +\sqrt{p}$. Then

$$f(-\sqrt{p}) = -p\sqrt{p} + 3p\sqrt{p} + 2 = 2p\sqrt{p} + 2 > 0 \text{ always}, \quad f(\sqrt{p}) = 2 - 2p\sqrt{p}.$$

We need $f(\sqrt{p}) < 0$, i.e. $p\sqrt{p} > 1$, i.e. $p > 1$. So the set is $p > 1$.

Examiner's eye. The case $p \leq 0$ must be dealt with first: with no stationary points the cubic has no turning points and so only one root. At $p = 1$ the polynomial is $(x-1)^2(x+2)$, a repeated root, so $p = 1$ is excluded.

Concavity. The second derivative measures how the gradient itself is changing. Where $f'' > 0$ the gradient is increasing and the curve bends upwards (**convex**: it lies below its chords and above its tangents). Where $f'' < 0$ it bends downwards (**concave**). A **point of inflection** is where f'' changes sign. This is exactly the information you need for the trapezium rule later in these notes, so remember it: *convex means chords above the curve*.

Worked example: when the second derivative is zero

Question. Let $f(x) = x^4 - 4x^3$. Find its stationary points and classify them, and the intervals where it is convex.

Solution. $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$, so the stationary points are at $x = 0$ and $x = 3$. $f''(x) = 12x^2 - 24x = 12x(x-2)$. At $x = 3$, $f'' = 36 > 0$, so a local minimum, with $f(3) = 81 - 108 = -27$. At $x = 0$, $f'' = 0$ is inconclusive, so look at $f' = 4x^2(x-3)$ on both sides: it is negative just left of 0 and negative just right of 0 (since $x^2 > 0$ and $x-3 < 0$). No sign change, so $x = 0$ is a stationary point of inflection, not a turning point. $f'' > 0$ for $x < 0$ or $x > 2$ (convex), and $f'' < 0$ for $0 < x < 2$ (concave).

Examiner's eye. The graph has a flat spot at the origin where it keeps going down. Deciding by $f'' = 0 \Rightarrow$ "minimum"

(or maximum) is a trap. The sign of f' on both sides never lies.

Reading the graph of f' , and what $f' > 0$ really says

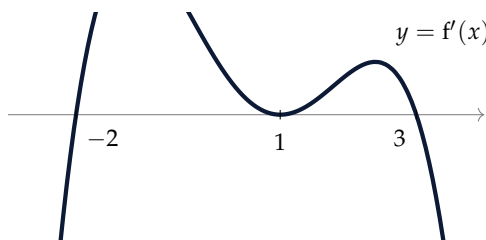
If you are given the *graph of f'* then you are really being given a sign chart for f . Where the graph of $y = f'(x)$ is above the x -axis, f is increasing; where below, decreasing. The points where it meets the axis are the stationary points of f , and what happens there tells you the type:

f' crosses the axis from below to above	f has a local minimum
f' crosses the axis from above to below	f has a local maximum
f' touches the axis and turns back	f has a stationary point of inflection , not a turning point

The heights of the graph of f' are *gradients* of f , not values of f , and you cannot read the value of f from them.

Worked example: which stationary points?

Question. The diagram shows $y = f'(x)$, a curve that crosses the x -axis at $x = -2$, touches it at $x = 1$ and crosses it at $x = 3$.



Describe the stationary points of f .

Solution. Read the sign of f' : it is negative for $x < -2$, positive on $(-2, 1)$, positive on $(1, 3)$ and negative for $x > 3$. So f decreases, then increases, then decreases. At $x = -2$, f' goes from $-$ to $+$: a **local minimum**. At $x = 3$, f' goes from $+$ to $-$: a **local maximum**. At $x = 1$, $f' = 0$ but the sign is $+$ on both sides, so f keeps increasing through a flat spot: a **stationary point of inflection**, neither maximum nor minimum.

Examiner's eye. The most common wrong answer calls $x = 1$ a minimum because the curve "touches" the axis there. The question is not what f' does near 1 in a geometric sense but whether it *changes sign*.

Worked example: the fraction of an interval

Question. Let $f(x) = 2x^3 - 3x^2 - 12x$ for $-3 \leq x \leq 5$. For what fraction of the interval $[-3, 5]$ is f decreasing?

Solution. $f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$. This is a positive x^2 parabola, so $f' < 0$ strictly between its roots: $-1 < x < 2$. That is a length of 3 inside an interval of length $5 - (-3) = 8$. The fraction is $\frac{3}{8}$.

Examiner's eye. The question asks about a *fraction of the domain*, so the answer is length of the decreasing part divided by length of the whole interval, with the interval given in the question (not the whole real line). If the problem hides f' in a factor such as $x^{1/7}$, the same idea works: find where each factor changes sign, using the *sign* of $x^{1/7}$ (same as the sign of x), and build a sign chart.

Increasing everywhere is not positive everywhere. If $f'(x) > 0$ for all x the function is strictly increasing, which says nothing about whether the values are positive: $f(x) = x$ has $f' = 1 > 0$ but $f(-5) < 0$. Likewise $f'(a) = 0$ does not mean a turning point (x^3 at 0). Whenever a question gives you a general "for a polynomial f " statement, test it with the simplest functions: x , x^2 , x^3 , a constant.

Worked example: which statements are always true?

Question. Let f be a polynomial. Consider: **I** if $f'(a) = 0$ then f has a local maximum or minimum at $x = a$; **II** if f has a local maximum at $x = a$ then $f'(a) = 0$; **III** if $f'(x) > 0$ for all x then $f(x) > 0$ for all x . Which of these must be true?

Solution. I is false: $f(x) = x^3$ has $f'(0) = 0$, yet $f(-0.1) < 0 < f(0.1)$, so 0 is neither a maximum nor a minimum. III

is false: $f(x) = x$ has $f' = 1 > 0$ but $f(-1) = -1 < 0$. II is true: a polynomial is smooth, and at the top of a smooth hill the tangent is horizontal. So **II only**.

Examiner's eye. This is the *Paper 2* version of the idea. For “which of I, II, III” questions decide each statement independently, and look for the simplest counterexample before trying to prove anything.

Tangents and normals

At the point $(a, f(a))$ on a curve, the **tangent** has gradient $f'(a)$ and equation $y - f(a) = f'(a)(x - a)$. The **normal** is the line through that point perpendicular to the tangent; its gradient is $-\frac{1}{f'(a)}$ (the product of perpendicular gradients is -1).

Two things trip people up. First, a statement such as “the line $y = mx + c$ is a normal to the curve at (p, q) ” gives you *three* separate facts: the point lies on the curve, the point lies on the line, and the line's gradient is $-1/f'(p)$. Write all three down. Second, a tangent drawn from an *external* point (one not on the curve) cannot be found by substituting that point into f' ; name the point of contact first.

Worked example: the line is a normal

Question. The line $x + 3y = k$ is the normal to the curve $y = x^2 - 3x + 4$ at the point (p, q) . Find p , q and k .

Solution. The line $y = -\frac{1}{3}x + \frac{k}{3}$ has gradient $-\frac{1}{3}$, so the tangent to the curve at (p, q) has gradient 3. Now $y' = 2x - 3$, so $2p - 3 = 3$ and $p = 3$. Then $q = 9 - 9 + 4 = 4$. The point $(3, 4)$ is on the line, so $k = 3 + 3 \cdot 4 = 15$. Answer: $p = 3$, $q = 4$, $k = 15$.

Examiner's eye. Writing the line in the form $y = mx + c$ first is what makes the gradient visible. A frequent slip is to use $+3$ as the normal gradient, or to forget that the tangent gradient is $-1/m$ and not $-m$.

Strategy: name the point of contact

To find a tangent to $y = f(x)$ through an external point, let the tangent touch the curve at $x = t$. The tangent there is $y = f(t) + f'(t)(x - t)$. Force it through the given point and solve the resulting equation for t . You usually get a quadratic, and its two roots are the two tangents. (A second method: let the line through the point have gradient m , equate it with the curve, and demand $\Delta = 0$.)

Worked example: tangents from an outside point

Question. Find the gradients of the two tangents to $y = x^2$ that pass through the point $(1, -3)$.

Solution. The tangent at $x = t$ is $y = t^2 + 2t(x - t) = 2tx - t^2$. It passes through $(1, -3)$ when $-3 = 2t - t^2$, i.e. $t^2 - 2t - 3 = 0$, so $t = 3$ or $t = -1$. The gradients are $2t$: 6 and -2 . **Check by the second method:** a line $y + 3 = m(x - 1)$ meets the curve where $x^2 - mx + m + 3 = 0$, with $\Delta = m^2 - 4m - 12 = (m - 6)(m + 2)$, which is zero at $m = 6$ and $m = -2$.

Examiner's eye. Two methods, one answer: this is how to check a tangent question in the exam. Note why there are two tangents: the point $(1, -3)$ is outside the parabola (below it), so two tangents can be drawn.

Common mistakes

- Using f' at the *external* point instead of at the point of contact.
- Using the tangent gradient for the normal (or $+1/m$ instead of $-1/m$).
- Forgetting that the point of tangency must lie on *both* the curve and the line.

Optimisation, end-points and rates of change

To find the greatest or least value of a quantity:

1. Write it as a function of **one** variable (use the given constraint to eliminate the others).
2. State the allowed range of that variable (lengths are positive, and so on).
3. Differentiate, solve $f' = 0$ and decide whether each solution is a maximum or minimum (sign change of f' or f'').
4. **Compare with the end-points** if the variable lives on a closed interval: the greatest value may be at an end, where f'

is not zero.

5. Answer the question that was asked: the optimal *value*, the optimal *position*, or the *distance*.

When the target is a **distance** to a point, minimise the *square* of the distance. The square root is largest or smallest at the same place, and you avoid a surd.

Strategy: minimise the square of the distance

If a point P moves as a parameter b changes, write OP^2 (or the distance squared to any fixed point) as a function of b . It is often a polynomial in b or in b^2 , which is far easier than differentiating a square root. Remember that $b^2 \geq 0$ restricts the substitution $u = b^2$.

Worked example: a stationary point that moves

Question. For each real b , the curve $y = x^2 - 2bx + 5$ has a stationary point P . Find the least possible distance from the origin O to P .

Solution. $\frac{dy}{dx} = 2x - 2b = 0$ gives $x = b$, and then $y = b^2 - 2b^2 + 5 = 5 - b^2$. So $P = (b, 5 - b^2)$ and

$$OP^2 = b^2 + (5 - b^2)^2 = b^4 - 9b^2 + 25 =: D(b).$$

Then $D'(b) = 4b^3 - 18b = 2b(2b^2 - 9)$, which is zero at $b = 0$ and $b^2 = \frac{9}{2}$. At $b = 0$, $D = 25$ (a local maximum of D); at $b^2 = \frac{9}{2}$, $D = \frac{81}{4} - \frac{81}{2} + 25 = \frac{19}{4}$. The smaller is $\frac{19}{4}$, and $D \rightarrow \infty$ as $|b| \rightarrow \infty$, so this is the least value. The least distance is $\sqrt{19/4} = \frac{\sqrt{19}}{2}$.

Examiner's eye. Notice the stationary point of D at $b = 0$ is a local *maximum*, not the answer. Always compare, don't just take the first root. A second route: let $u = b^2 \geq 0$; then $D = u^2 - 9u + 25$ is a quadratic with minimum at $u = \frac{9}{2}$, which is allowed since $u \geq 0$.

Worked example: a box with a fixed volume

Question. An open-topped tank has a square base of side x metres and vertical sides, and holds exactly 32 m^3 . Find the least possible area of sheet metal needed, and the height of that tank.

Solution. Height $h = \frac{32}{x^2}$. The metal area is the base plus four sides: $S = x^2 + 4xh = x^2 + \frac{128}{x}$. Then $S' = 2x - \frac{128}{x^2} = 0$ gives $x^3 = 64$, $x = 4$. $S'' = 2 + \frac{256}{x^3} > 0$, so a minimum, and $S = 16 + 32 = 48$. The height is $\frac{32}{16} = 2$. The least area is 48 m^2 , with height 2 m .

Examiner's eye. A hidden 2-for-1: at the optimum the base area 16 is half of the side area 32. Spotting such a relation tells you instantly whether your answer is plausible. Using $S'' > 0$ here is easy; on a closed interval you would compare with the end-points.

Worked example: a closed interval: check the end-points

Question. The volume of water in a tank is $V(t) = t^3 - 6t^2 + 9t$ litres at time t hours, for $0 \leq t \leq 5$. Find the greatest and least volumes, and when V is decreasing.

Solution. $V'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$. So V increases on $(0, 1)$, decreases on $(1, 3)$ and increases on $(3, 5)$. Candidates: the turning points $V(1) = 4$ and $V(3) = 0$, and the end-points $V(0) = 0$ and $V(5) = 125 - 150 + 45 = 20$. The greatest volume is 20 litres (at $t = 5$, an *end-point*, not a turning point) and the least is 0 (at $t = 0$ and $t = 3$). V is decreasing for $1 < t < 3$.

Examiner's eye. If you had only looked at the stationary points you would have said the greatest volume is 4. The greatest value on a closed interval is at a turning point *or* an end-point. Always list both and compare.

Common mistakes

- Giving the *position* x of the optimum when the question wants the optimal *value* (or vice versa).
- Forgetting the end-points when the domain is a closed interval.
- Differentiating the distance (with its square root) rather than the squared distance.

Habits that win marks on differentiation questions

- **Sketch the cubic.** For any question about roots of a cubic, draw it with its two turning points on the correct sides of the axis. The inequalities then write themselves.
- **Rewrite before you differentiate.** Surds and quotients become powers of x . Convert back only at the end.
- **Check the sign change, not just $f' = 0$.** A stationary point is not automatically a maximum or minimum. Look either side, or use f'' only when it is non-zero.
- **Test a simple function.** Statements about “all functions” are quickly refuted with x , x^2 or x^3 .
- **Minimise the square** for distances, and list end-points as well as turning points on a closed interval.
- **Check with a second method.** Tangent via the discriminant and via the derivative; roots of a cubic via a sketch and via the stationary values. If the two agree, move on.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Find $\frac{dy}{dx}$ at $x = 1$ for $y = \frac{x^3 - 2}{x\sqrt{x}}$.
2. Find the stationary points of $y = x^3 - 3x^2 - 9x + 2$ and say which is the local maximum.
3. Find the values of k for which $x^3 - 3x^2 - 9x + k = 0$ has three distinct real roots.
4. Find where the normal to $y = x^2 + 1$ at $x = 1$ meets the y -axis.
5. $f'(x) = x(x - 2)^2$. Classify the stationary points of f .
6. Find the minimum value of $x^2 + \frac{16}{x}$ for $x > 0$.
7. True or false, for a polynomial f with $f'(x) > 0$ for all real x : (a) $f(x) = 0$ has at most one real root; (b) $f(x) = 0$ has at least one real root?
8. Show that $y = x^3 - 6x^2 + 12x + 1$ has no turning points.

Answers to the self-check

1. $y = x^{3/2} - 2x^{-3/2}$, so $y' = \frac{3}{2}x^{1/2} + 3x^{-5/2}$. At $x = 1$: $\frac{3}{2} + 3 = \frac{9}{2}$.
2. $y' = 3(x-3)(x+1)$. At $x = -1$: $y = -1 - 3 + 9 + 2 = 7$, the local maximum. At $x = 3$: $y = -25$, the local minimum.
3. Stationary values: at $x = -1, 5 + k$ (max); at $x = 3, k - 27$ (min). Need $5 + k > 0 > k - 27$, so $-5 < k < 27$.
4. Point $(1, 2)$, tangent gradient 2, normal gradient $-\frac{1}{2}$: $y - 2 = -\frac{1}{2}(x - 1)$. At $x = 0$: $y = \frac{5}{2}$.
5. Sign of f' : negative for $x < 0$, positive on $(0, 2)$ and for $x > 2$. So $x = 0$ is a minimum, and $x = 2$ is a stationary point of inflection (f' does not change sign).
6. $2x - \frac{16}{x^2} = 0$ gives $x^3 = 8$, $x = 2$. The value is $4 + 8 = 12$, a minimum since the second derivative $2 + \frac{32}{x^3} > 0$.
7. (a) True: f is strictly increasing, so it takes each value at most once. (b) True for a polynomial (odd degree, because $f' > 0$ means f climbs from $-\infty$ to $+\infty$). Without the word "polynomial" (b) would be false, as for 2^x .
8. $y' = 3x^2 - 12x + 12 = 3(x-2)^2 \geq 0$, which is zero only at $x = 2$ without changing sign. So the only stationary point is a point of inflection.

Next step

Now do the matching topic practice set: 20 questions in TMUA style, with worked solutions. Mark it, then redo the ones you missed a week later without looking at the solutions.