

TMUA

Paper 1

Graphs and transformations

Free lesson notes

Covers	1.8 graphs of functions, transformations, number of solutions
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

A good sketch answers a surprising fraction of TMUA questions in a few seconds. “How many solutions does this equation have?” becomes “how many times do these two graphs meet?”. “Which of these is the equation of the curve?” becomes “which one has the right intercepts and asymptotes?”. “What can you say about $f(x + 1) = 0$?” becomes “what does moving the graph do to the roots?”. A sketch does not need to be pretty. It needs to show the *features that decide the answer*: where it crosses the axes, where it turns, what it does at large $|x|$, and any asymptotes.

This lesson teaches you to sketch quickly from the algebra, to apply transformations in the correct order without mistakes, to handle the modulus, and to count solutions by intersecting graphs. There are twelve worked examples, each tied to a pattern that recurs, and each was checked by computer as well as by hand.

How to use this lesson. Read a section, then cover each worked example and try it before reading the solution. The TMUA gives you about 90 seconds a question on average, so after every idea ask two things: what is the quickest sound method, and what would a quick sketch or a test value tell me?

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The patterns to expect

What the TMUA keeps asking

- **Which graph is it?** A sketch is shown (often a rational or exponential curve such as $y = \frac{2^x}{1+2^x}$) and you must choose the matching equation by testing intercepts, asymptotes and behaviour for large $|x|$.
- **Sequences of transformations:** stretch, translate and reflect in a stated order, and say what the equation (or a vertex, or a root) becomes. The order matters.
- **How many solutions?** An equation mixing different families, such as $3 \cos x = \sqrt{x}$ or $2^x = x^2$, to be solved by sketching both sides. Or $|x^2 - a| = k$ for which k the number of solutions changes.
- **Symmetry conditions:** which of $f(a - x) = f(a + x)$, $f(2a - x) = f(x)$, $f(x - a) = f(x + a)$ expresses a mirror line, and what that implies for the roots.
- **Roots and transformations:** if $f(x) = 0$ has three distinct roots, what can you say about $f(x) + 1 = 0$, $f(x + 1) = 0$, $f(-x) - 1 = 0$?
- **Parameters in a vertex:** the vertex of $y = x^2 - 2bx + c$ and how it moves as b changes.
- **Modulus graphs:** $|f(x)|$ and $f(|x|)$, and inequalities such as $|x - 2| + |x + 1| \leq 5$.

Almost none of these needs a precise plot. They need a sensible one, drawn in under half a minute.

Sketching polynomials from their factors

To sketch a polynomial you need four pieces of information, all of which come from the factorised form:

1. the **roots** (where each factor is zero);
2. the **multiplicity** of each root. A simple root (power 1) means the graph *crosses* the axis. A double root (power 2, or any even power) means it *touches and turns back*. A triple root (odd power) means it crosses with a flattened, S-shaped bend;
3. the **end behaviour**: the sign of the leading coefficient and the degree. Positive leading coefficient: $+\infty$ as $x \rightarrow +\infty$. For even degree the left end goes the same way as the right; for odd degree it goes the opposite way;
4. the **y-intercept**, found by putting $x = 0$.

The sign of the polynomial changes at a root exactly when its multiplicity is odd. So you can read off where the polynomial

is positive or negative by walking along the axis from the right-hand end, flipping sign at each root of odd multiplicity and keeping it at each root of even multiplicity.

Strategy: walk the number line from the right

Start at the far right with the sign of the leading coefficient. Move left past each root in turn: flip the sign if the multiplicity is odd, keep it if even. Write the sign on each stretch. For " ≥ 0 " inequalities, include the roots themselves, and remember an isolated double root satisfies " ≥ 0 " even when both neighbours are negative.

Worked example: an inequality with a repeated root

Question. Solve $x^2(x-1)(x+2)^3 \geq 0$.

Solution. The roots are $x = 0$ (double), $x = 1$ (simple) and $x = -2$ (triple). The leading coefficient is $+1$, so the polynomial is positive for $x > 1$. Walking left: crossing $x = 1$ (odd) flips to negative on $(0, 1)$; crossing $x = 0$ (even) keeps negative on $(-2, 0)$; crossing $x = -2$ (odd) flips to positive for $x < -2$.

$$x < -2: + \quad -2 < x < 0: - \quad 0 < x < 1: - \quad x > 1: +$$

The polynomial is ≥ 0 where it is positive or zero: $x \leq -2$, or $x = 0$, or $x \geq 1$.

Examiner's eye. The isolated solution $x = 0$ is the classic marks-loser: the graph touches the axis there from below, and zero satisfies " ≥ 0 ". Had the question said " > 0 ", the answer would be $x < -2$ or $x > 1$.

Rational, exponential and sigmoid graphs

For **rational functions** $y = \frac{\text{linear}}{\text{linear}}$ the four features are: the **vertical asymptote** (denominator zero), the **horizontal asymptote** (ratio of the leading coefficients as $x \rightarrow \pm\infty$), the **x-intercept** (numerator zero) and the **y-intercept** (put $x = 0$). Plot those and the shape is a hyperbola in two pieces. For **exponentials** $y = a^x$ ($a > 1$) the graph passes through $(0, 1)$, is always positive, grows rapidly to the right and approaches the x -axis on the left. Logarithms are the reflection in $y = x$.

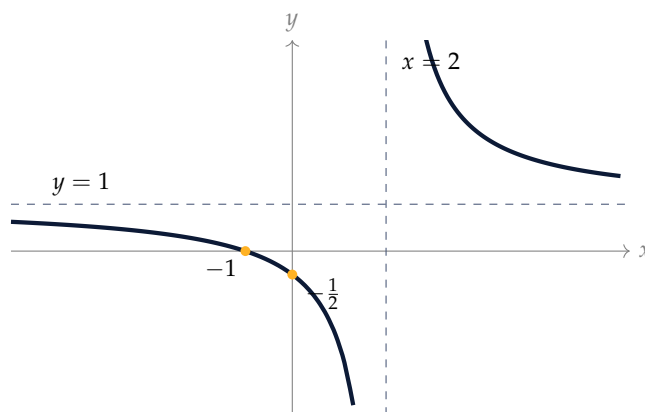
When asked to **match a graph to an equation**, do not guess from the general look. Test the facts in the diagram one at a time against each option, starting with whichever fact eliminates the most candidates.

Strategy: eliminate with features

Pick the most informative feature of the diagram (an asymptote, an intercept, the sign at a test point, the limit for large x) and cross off every option that fails it. Two features usually leave one option. If an option survives both, check a third before you commit.

Worked example: matching a rational curve

Question. The diagram shows a curve with a vertical asymptote $x = 2$ and a horizontal asymptote $y = 1$, passing through $(-1, 0)$ and $(0, -\frac{1}{2})$.



Which is its equation? **A** $y = \frac{x+1}{x-2}$ **B** $y = \frac{x-1}{x+2}$ **C** $y = \frac{x+1}{2-x}$ **D** $y = \frac{x-2}{x+1}$ **E** $y = \frac{x+1}{x+2}$.

Solution. The vertical asymptote is where the denominator is zero, $x = 2$. That rules out B and E (denominator zero

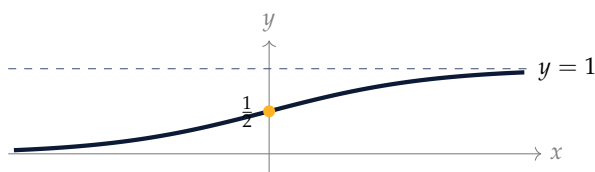
at -2) and D (zero at -1). Of A and C, the horizontal asymptote $y = 1$ rules out C, which tends to -1 for large $|x|$. Check A: it is zero when $x = -1$ and equals $-\frac{1}{2} = -\frac{1}{2}$ at $x = 0$. The answer is A.

Examiner's eye. Notice that testing $x = 0$ alone would have left both A and B (each gives $-\frac{1}{2}$). Always use more than one feature, and prefer the asymptote: it is the quickest eliminator.

The S-shaped (sigmoid) curve. The function $f(x) = \frac{2^x}{1+2^x}$ is a TMUA favourite. It is positive. For large positive x the 1 in the denominator is negligible, so $f \rightarrow 1$ from below. For large negative x , $2^x \rightarrow 0$ so $f \rightarrow 0$. At $x = 0$ it equals $\frac{1}{2}$. It is increasing throughout. Most usefully,

$$f(-x) = \frac{2^{-x}}{1+2^{-x}} = \frac{1}{2^x+1} = 1 - f(x),$$

so the graph is symmetric under a half turn about the point $(0, \frac{1}{2})$: $f(x) + f(-x) = 1$.



Worked example: using the symmetry

Question. Let $f(x) = \frac{2^x}{1+2^x}$. Find $f(-3) + f(-2) + f(-1) + f(0) + f(1) + f(2) + f(3)$.

Solution. Since $f(x) + f(-x) = 1$, the terms pair up: $f(3) + f(-3) = 1$, likewise for ± 2 and ± 1 , giving 3. The middle term is $f(0) = \frac{1}{2}$. The total is $3\frac{1}{2} = \frac{7}{2}$.

Examiner's eye. Evaluating seven fractions would take minutes and invite arithmetic errors. The graph's symmetry about $(0, \frac{1}{2})$ does it in seconds. Whenever a sum has terms symmetric about zero, look for a pairing identity.

Transformations, in the stated order

Starting from $y = f(x)$:

$y = f(x) + a$	translation a units up (vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$)
$y = f(x + a)$	translation a units to the left (vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$)
$y = af(x)$	stretch in the y -direction by factor a
$y = f(ax)$	stretch in the x -direction by factor $\frac{1}{a}$ (so $f(\frac{x}{2})$ is a stretch by factor 2)
$y = -f(x)$	reflection in the x -axis
$y = f(-x)$	reflection in the y -axis

Changes *outside* the function $(+a, a\times, -)$ act on y and do what you expect. Changes *inside* the brackets act on x and do the opposite of what you expect: $+a$ moves left, and $\times a$ shrinks by a .

To find the equation after several transformations **do them one at a time**. After each transformation, you have a new equation $y = \dots$. The next transformation then acts on that equation: "stretch by factor 2 in the x -direction" means replace x by $\frac{x}{2}$ *everywhere* x appears in the current equation; "translate 3 to the right" means replace x by $x - 3$ everywhere in the current equation. Do not try to combine the steps in your head.

Strategy: one step, one substitution

Write the current equation. Apply the next transformation by substituting into *that* equation, simplify, and write the result. Repeat. Afterwards test with a single point: track $(u, f(u))$ through each step and check it lies on your final equation.

Worked example: three steps, and the order matters

Question. The curve $y = f(x)$, where $f(x) = x^2 + 1$, is (i) stretched by factor 2 in the x -direction, then (ii) translated 3 units in the positive x -direction, then (iii) reflected in the x -axis. Find the final equation. What would the equation be if steps (i) and (ii) were done in the opposite order?

Solution. (i) Replace x by $\frac{x}{2}$: $y = (\frac{x}{2})^2 + 1$. (ii) Replace x by $x - 3$: $y = (\frac{x-3}{2})^2 + 1$. (iii) Negate: $y = -\frac{(x-3)^2}{4} - 1$, a maximum at $(3, -1)$.

In the opposite order: translate first, $y = (x - 3)^2 + 1$; then stretch (replace x by $\frac{x}{2}$): $y = (\frac{x}{2} - 3)^2 + 1$; then reflect: $y = -\frac{(x-6)^2}{4} - 1$, a maximum at $(6, -1)$. The vertex has moved to a different place.

Examiner's eye. Check with a point: the vertex $(0, 1)$ of the original goes to $(0, 1) \rightarrow (0, 1) \rightarrow (3, 1) \rightarrow (3, -1)$ in the first order, and $(0, 1) \rightarrow (3, 1) \rightarrow (6, 1) \rightarrow (6, -1)$ in the second. A stretch moves points *away from the y -axis*, so doing it after the shift doubles the shift.

Worked example: follow the turning point

Question. The curve $y = f(x)$ has a minimum point at $(2, 5)$. Find the coordinates and the nature of the stationary point of (a) $y = 3 - 2f(x - 1)$ and (b) $y = f(\frac{x}{3} + 1)$.

Solution. (a) $f(x - 1)$ translates by 1 to the right: the minimum moves to $(3, 5)$. Multiplying by -2 stretches by factor 2 vertically and reflects, so a minimum becomes a maximum, with y -value $-2 \times 5 = -10$. Adding 3 lifts it: $-10 + 3 = -7$. So a **maximum at $(3, -7)$** . (b) The turning point of f occurs when the bracket equals 2: $\frac{x}{3} + 1 = 2$, so $x = 3$. The y -value is unchanged, so a **minimum at $(3, 5)$** .

Examiner's eye. For “inside” changes use the shortcut: set the bracket equal to the old x -coordinate and solve for the new x . It is quicker and safer than thinking about stretches and shifts.

Worked example: what happens to the roots?

Question. f is a cubic polynomial and $f(x) = 0$ has three distinct real roots. Which of the following must be true? **I** $f(x) + 1 = 0$ has at least one real root. **II** $f(x) + 1 = 0$ has exactly three distinct real roots. **III** $f(x + 1) = 0$ has exactly three distinct real roots.

Solution. III is true: $y = f(x + 1)$ is the graph shifted one unit left, and shifting does not change how many times the curve meets the x -axis. II is not always true: $f(x) = x^3 - x$ has three roots, but $x^3 - x + 1 = 0$ has only one (the graph shifted up has its local minimum $1 - \frac{2}{3\sqrt{3}} > 0$). I is true: a cubic goes from $-\infty$ to $+\infty$, so it takes every value, in particular -1 . So **I and III only**.

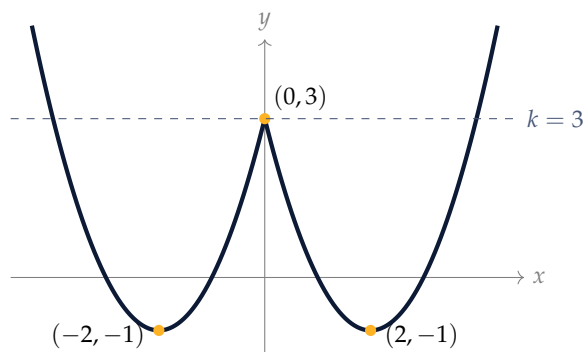
Examiner's eye. Translating along the axis never changes the *number* of roots; translating up or down can. Statements about “ $f(x) + c = 0$ ” depend on how far the turning points are from the axis, so “exactly three” fails in general, while “at least one” holds for any cubic.

Modulus graphs

The modulus “folds” a graph. There are two different folds, and they are easy to confuse.

- $y = |f(x)|$: keep the parts of the graph on or above the x -axis, and **reflect the parts below the axis upwards**. The result lies entirely on or above the axis.
- $y = f(|x|)$: keep the part of the graph with $x \geq 0$, discard the rest, and **reflect the right half in the y -axis**. The result is always symmetric about the y -axis (it is an even function).

To count solutions of $|f(x)| = k$ or $f(|x|) = k$, sketch the folded graph and slide a horizontal line $y = k$ up the page, counting crossings. The count changes only when the line passes through a turning point, a corner, or an intercept, so you only need to check those special values of k and one value between each.



Worked example: sliding a line

Question. For which values of k does $x^2 - 4|x| + 3 = k$ have exactly (a) three solutions; (b) two solutions?

Solution. Let $f(x) = x^2 - 4|x| + 3 = (x - 2)^2 - 1$, which has a minimum $(2, -1)$ and y -intercept 3. The curve $y = f(|x|)$ keeps the right half and reflects it: a W shape with minima $(\pm 2, -1)$ and a central local maximum $(0, 3)$ (see the diagram). Sliding the line $y = k$ upwards:

$$k < -1: 0 \quad k = -1: 2 \quad -1 < k < 3: 4 \quad k = 3: 3 \quad k > 3: 2$$

(a) Exactly three: $k = 3$ (the line passes through the central peak). (b) Exactly two: $k = -1$ or $k > 3$.

Examiner's eye. The three-solutions case is the tell-tale: three is odd, so one of the solutions must lie on the axis of symmetry ($x = 0$). Check: $0 - 0 + 3 = 3$. Odd numbers of solutions for an even function always include $x = 0$.

Worked example: a sum of distances

Question. Solve $|x - 2| + |x + 1| \leq 5$.

Solution. Read $|x - 2|$ as the distance from x to 2 and $|x + 1|$ as the distance from x to -1 . For x between -1 and 2 the two distances add to 3 (the gap between the points). Moving outside the gap by a distance d adds $2d$ to the sum. For the sum to equal 5 we need $2d = 2$, so $d = 1$: the endpoints are $x = -2$ and $x = 3$. The graph of the left-hand side is flat at height 3 between -1 and 2, rising with gradient ± 2 outside, so the inequality holds for $-2 \leq x \leq 3$.

Examiner's eye. The algebraic route needs three cases (below -1 , between, above 2). The distance picture needs none. Whenever you see a sum of moduli of linear expressions, picture the graph: it is piecewise linear, flat or sloping, with corners at the points where each modulus changes.

Counting solutions by intersecting graphs

When an equation mixes function types, such as $2^x = x^2$ or $2 \sin x = x$, it cannot be solved by algebra. It can be *counted* by sketching each side as a separate graph and counting intersections. The method:

1. Rewrite the equation as $g(x) = h(x)$, with both sides as simple as possible (put the awkward bit on one side alone).
2. Sketch both. Include the features that decide whether they can meet: the range of each (for example $|\sin x| \leq 1$), where each is increasing or decreasing, and the values at easy points.
3. Count the crossings in each region. Use *test values* at the ends of each region to be sure whether a crossing happens (the graphs swap order).
4. Argue that there are no more: one graph is above the other for all larger $|x|$, or the range of one side is too small.

Strategy: compare at easy points

After sketching, evaluate both sides at a few easy points to establish which is bigger, and find where they swap. A crossing exists between two points exactly when the order of the two graphs differs at those points (for continuous functions). That turns a vague picture into a proof.

Worked example: exponential against a parabola

Question. How many real solutions does $2^x = x^2$ have?

Solution. For $x > 0$: $x = 2$ and $x = 4$ are solutions ($4 = 4$ and $16 = 16$). Test the order of the two graphs around

them: at $x = 1$, $2^1 = 2 > 1 = 1^2$ (exponential above); at $x = 3$, $8 < 9$ (parabola above); at $x = 5$, $32 > 25$ (exponential above, and from here on it grows faster). So the graphs cross at 2 and at 4, and there are no other positive crossings. For $x \leq 0$: at $x = 0$, $2^0 = 1 > 0$ (exponential above); at $x = -1$, $2^{-1} = \frac{1}{2} < 1$ (parabola above). The order swaps between -1 and 0 , so there is one crossing there. For $x < -1$ the exponential is below 1 while $x^2 > 1$, so no more. Total: **three solutions**.

Examiner's eye. Most students find 2 and 4 and stop at two, forgetting the negative root that only a sketch reveals. The exact values at 2 and 4 are the giveaway for the positive roots. (The negative root is irrational, near -0.77 , and you do not need it.)

Worked example: a trigonometric graph against a line

Question. How many real solutions does $2 \sin x = x$ have? (Angles in radians.)

Solution. Write it as $\sin x = \frac{x}{2}$ and sketch the sine curve and the line through the origin of gradient $\frac{1}{2}$. Both are odd, so solutions come in pairs $\pm$, plus $x = 0$. For $x > 0$: near 0 the sine curve has gradient $1 > \frac{1}{2}$, so it starts above the line. At $x = \frac{\pi}{2}$, $\sin x = 1 > \frac{\pi}{4}$ (the line is still below), and at $x = \pi$, $\sin x = 0 < \frac{\pi}{2}$. So the order swaps once on $(\frac{\pi}{2}, \pi)$: exactly one crossing there, and none between 0 and $\frac{\pi}{2}$ (the sine is concave and starts above the line). For $x > 2$ the line is above $1 \geq \sin x$, so there are no more. Positive solutions: 1; negative: 1; and $x = 0$. Total **three**.

Examiner's eye. The key observation is the gradient comparison at the origin (1 against $\frac{1}{2}$) and the range bound $|\sin x| \leq 1$ that stops any crossing beyond $x = 2$. With a gradient of 1 or more for the line the origin would be the only solution; with gradient $\frac{1}{10}$ there would be more humps to count (seven solutions in all).

Worked example: periodic graphs and the modulus

Question. How many solutions does $|\cos 2x| = \frac{1}{2}$ have for $0 \leq x < 2\pi$?

Solution. The graph of $y = \cos 2x$ has period π , so over $[0, 2\pi)$ it has two full cycles. Taking the modulus reflects the negative parts upwards, giving four arches of period $\frac{\pi}{2}$, each peaking at height 1 and touching the axis. The line $y = \frac{1}{2}$ crosses each arch twice. So there are $4 \times 2 = 8$ solutions. (Algebraically: $\cos \theta = \pm \frac{1}{2}$ has four solutions in each interval of length 2π , and $\theta = 2x$ runs over $[0, 4\pi)$, which is two such intervals: $2 \times 4 = 8$.)

Examiner's eye. Count arches, not solutions: the sketch of $|\cos 2x|$ makes the answer visible. Stretches inside the argument ($2x$) multiply the number of cycles by 2, which is the usual way an easy-looking trig question becomes a TMUA question.

Common mistakes

- Stopping at the solutions you can find exactly (2 and 4) and forgetting solutions that exist but have no nice value.
- Counting crossings of the graphs without checking that you have the right *range* of x (negative x , large x , the interval stated in the question).
- Treating $|f(x)| = k$ as having the same number of solutions as $f(x) = k$, forgetting both $f(x) = k$ and $f(x) = -k$.

Symmetry of graphs

A graph is **symmetric about the vertical line** $x = a$ when every point at distance s to the left of the line has the same height as the point at distance s to the right:

$$f(a - x) = f(a + x) \text{ for all } x.$$

Writing $a + x$ as u gives the equivalent form $f(2a - u) = f(u)$ for all u : the reflection of u in a is $2a - u$. These are the *same condition*, written differently. Do not confuse it with the other two relatives:

- $f(-x) = f(x)$ is the special case $a = 0$ (even function). $f(-x) = -f(x)$ is half-turn symmetry about the origin (odd).
- $f(x + a) = f(x)$ says the graph *repeats* every a units (periodic), which is a different thing.

Symmetry about $x = a$ means that roots come in mirror pairs r and $2a - r$ (unless $r = a$, in which case it is its own mirror), so the roots average to a . For a quadratic $y = x^2 - 2bx + c$ the axis of symmetry is $x = b$, the vertex is $(b, c - b^2)$, and increasing b moves the vertex to the right.

Worked example: which condition means a mirror line?

Question. (a) The graph of $y = f(x)$ is symmetrical about the line $x = 2$. Which of the following must hold for all x ? **I** $f(2 + x) = f(2 - x)$; **II** $f(4 - x) = f(x)$; **III** $f(x - 2) = f(x + 2)$. (b) A quartic polynomial p satisfies $p(3 + x) = p(3 - x)$ for all x and has four distinct real roots, two of which are 1 and 4. Find the sum of its roots.

Solution. (a) I is the definition with $a = 2$. II is the same statement with $a = 2$ in the form $f(2a - x) = f(x)$. III says $f(u) = f(u + 4)$, i.e. period 4, which is not a mirror condition: for $f(x) = (x - 2)^2$ (symmetric about $x = 2$) we get $f(x - 2) = (x - 4)^2$ but $f(x + 2) = x^2$, which differ. So **I and II only**. (b) The roots pair up about $x = 3$: the mirror of 1 is $2 \cdot 3 - 1 = 5$ and the mirror of 4 is 2. These are four distinct roots, so they are all of them: 1, 2, 4, 5, with sum 12 ($= 4 \times 3$).

Examiner's eye. For III the quickest refutation is the simplest symmetric function, $(x - 2)^2$. For (b) the shortcut is that the roots average to the axis of symmetry, giving $4 \times 3 = 12$ without even finding the other two roots individually.

Habits that win marks on graph questions

- **Sketch the feature list first:** intercepts, turning points, asymptotes, end behaviour. Label the values; do not draw a pretty curve with no numbers on it.
- **Apply transformations one at a time**, writing the new equation after each step. Check the result by following one point through.
- **For inside changes, solve for the old x :** set the bracket equal to the old coordinate and solve for the new one.
- **Count by intersecting:** one graph on each side, then compare at easy values to see where they swap order. Look for solutions on *both* sides of zero and for "hidden" ones.
- **Match by eliminating:** use the asymptote first, then an intercept, then large $|x|$. Never decide by one point alone.
- **Know the three symmetry statements:** $f(a - x) = f(a + x)$ and $f(2a - x) = f(x)$ are the same mirror line; $f(x + a) = f(x)$ is periodicity.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Solve $(x - 1)^2(x + 2) < 0$.
2. The vertex of $y = f(x)$ is $(1, -2)$. Where is the vertex of $y = 2f(x + 3) + 1$?
3. The graph of $y = f(x)$ is stretched by factor 3 in the y -direction and then reflected in the y -axis. Find the equation.
4. How many solutions has $|x^2 - 9| = 5$?
5. How many real solutions has $\sin x = \frac{x}{10}$?
6. $f(x) = 0$ has roots 1, 2, 5. Find the roots of $f(\frac{x}{2}) = 0$.
7. The graph of $y = f(x)$ is symmetric about $x = -1$ and $f(3) = 0$. Name another root.
8. Solve $|x + 1| + |x - 3| = 6$.

Answers to the self-check

1. The sign changes only at the simple root -2 (positive for $x > -2$ apart from the touch at 1). So the product is negative only for $x < -2$.
2. Shift left 3: $(-2, -2)$. Then y -stretch by 2 and shift up 1: $-2 \times 2 + 1 = -3$. Vertex $(-2, -3)$.
3. Stretch: $y = 3f(x)$. Reflect in the y -axis (replace x by $-x$): $y = 3f(-x)$.
4. $x^2 - 9 = 5$ gives $x^2 = 14$ (two solutions); $x^2 - 9 = -5$ gives $x^2 = 4$ (two solutions). Total 4.
5. Both sides are odd, so count positive solutions and double, then add $x = 0$. The line reaches 1 at $x = 10$. Positive solutions: one in $(0, \pi)$, and two in $(2\pi, 3\pi)$ (the line is at about 0.6 to 0.9, below the peak 1). The hump $(4\pi, 5\pi)$ has the line above 1, so none. That is 3 positive, 3 negative, and 0: total 7.
6. $\frac{x}{2} = 1, 2, 5$, so $x = 2, 4, 10$ (a stretch by factor 2 in the x -direction).
7. The mirror of 3 in $x = -1$ is $2(-1) - 3 = -5$. So $f(-5) = 0$.
8. The sum of distances from -1 and 3 is 4 between them. To make it 6 go 1 outside each end: $x = -2$ or $x = 4$.

Next step

Now do the matching topic practice set: 20 questions in TMUA style, with worked solutions. Mark it, then redo the ones you missed a week later without looking at the solutions.