

TMUA

Paper 1

Integration

Free lesson notes

Covers	1.7 integration, areas, the trapezium rule
Format	Lesson, worked examples, self-check
Price	Free

About this lesson

Integration questions on the TMUA are rarely long calculations. They test whether you can tell a *signed* integral from an *area*, whether you can use a symmetry, a shift or a geometric shape to avoid integrating at all, and whether you can reason about the trapezium rule from the shape of a curve instead of computing it. A common feature of the real papers is that the question gives you a few integral values over certain intervals and asks for something different, and the whole trick is to see how the pieces combine.

This lesson teaches the integration facts you need (powers, rewriting, areas, symmetry, shifts, variable limits, the trapezium rule) together with the strategies that save time. There are twelve worked examples, each tied to a pattern that recurs, and each was checked by computer as well as by hand. If you take one idea away, make it this: *sketch the curve before you integrate it*.

How to use this lesson. Read a section, then cover each worked example and try it before reading the solution. The TMUA gives you about 90 seconds a question on average, so after every idea ask two things: what is the quickest sound method, and what would a quick sketch or a test value tell me?

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The patterns to expect

What the TMUA keeps asking

- **Definite integrals of powers after rewriting:** a quotient with a surd or a power of x in the denominator, split into terms and integrated using fractional or negative powers.
- **Area between a line and a curve,** or between two curves, where one curve contains a parameter (m or p) that you must find from the given area. Symmetry usually halves the work.
- **Regions of equal area** for a curve with roots at 2 and q : equal areas above and below the axis means the signed integral is zero.
- **Signed integral against total area:** a function that crosses the axis, where you are *given* the integrals over some sub-intervals and must find a different one. Split the interval at the roots.
- **Odd and even functions, modulus, and geometry:** integrals involving $|x|$, $f(|x|)$, or a region such as $|x| + |y| = 1$ whose area you read off a diagram.
- **Shifts and stretches:** $\int f(x+1) dx$, $\int f(2x) dx$, and a sum of integrals over adjacent unit intervals that collapses into a single integral.
- **Variable limits:** the derivative of $\int_x^{2x} t^2 dt$, or a sequence defined by an integral.
- **The trapezium rule:** the estimate with n strips, and whether it is an over- or under-estimate, argued from convexity and extended to transformed integrands.

Notice how many of these never ask you to find an antiderivative of anything difficult. They ask you to *think* about what an integral means.

Integrating powers, after rewriting

For any $n \neq -1$,

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad \int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a),$$

where F is any antiderivative. As in differentiation, the whole skill is rewriting. Convert roots and reciprocals to powers, split a quotient into one term per numerator term, and integrate term by term. Three habits make the arithmetic safe:

- *Raise the power by one, then divide by the new power.* For $x^{-1/2}$ the new power is $\frac{1}{2}$, so $\int x^{-1/2} dx = 2x^{1/2}$.

- Evaluate at the upper limit, then the lower, then subtract. Keep brackets: $F(4) - F(1)$ with a negative $F(1)$ is where signs go astray.
- Choose the limit that makes powers easy. $x^{3/2}$ and $x^{5/2}$ at $x = 4$ or 9 become whole numbers.

Worked example: surds and negative powers

Question. Evaluate (a) $\int_1^4 \frac{x^2+3}{\sqrt{x}} dx$ and (b) $\int_1^2 \frac{2x^3+1}{x^2} dx$.

Solution. (a) Split: $\frac{x^2+3}{x^{1/2}} = x^{3/2} + 3x^{-1/2}$. An antiderivative is $\frac{2}{5}x^{5/2} + 6x^{1/2}$. At $x = 4$: $\frac{2}{5} \cdot 32 + 6 \cdot 2 = \frac{64}{5} + 12 = \frac{124}{5}$. At $x = 1$: $\frac{2}{5} + 6 = \frac{32}{5}$. The integral is $\frac{124}{5} - \frac{32}{5} = \frac{92}{5}$.

(b) Split: $2x + x^{-2}$, with antiderivative $x^2 - x^{-1}$. At $x = 2$: $4 - \frac{1}{2} = \frac{7}{2}$. At $x = 1$: $1 - 1 = 0$. The integral is $\frac{7}{2}$.

Examiner's eye. Both answers come from the same three moves: split, convert to powers, integrate. Never integrate the numerator and denominator separately (a classic error: $\int \frac{f}{g} \neq \frac{\int f}{\int g}$).

Signed integrals, total area and equal areas

A definite integral is a **signed** area: regions above the x -axis count positively and regions below count negatively. So $\int_a^b f(x) dx$ is *not* the area between the curve and the axis when the curve crosses the axis between a and b . To find the **total area**, split at every root in (a, b) and add the *positive sizes* of the pieces:

$$\text{total area} = (\text{signed integral}) + 2 \times (\text{size of the part below the axis}).$$

The second form is useful when you are *given* partial integrals and cannot (or need not) compute them. Always start by working out the sign of the curve on each interval: that tells you which given values are areas and which are the negative of areas.

Strategy: sign chart, then split

Sketch (or just factorise) to find where f is positive and negative. Mark the roots on a line and write $+/-$ on each stretch. Then every integral over a stretch is $\pm$ an area, and the total area is the sum of the absolute values. If you are given integrals over *some* sub-intervals, the unknown one can usually be found by adding and subtracting: $\int_0^3 = \int_0^1 + \int_1^2 + \int_2^3$.

Worked example: given partial integrals

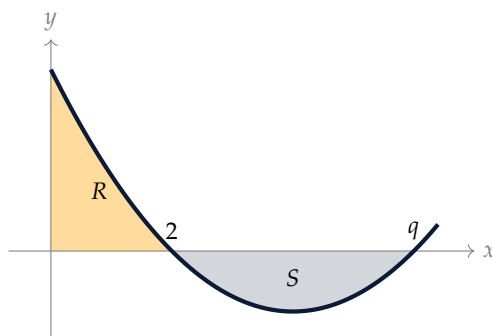
Question. Let $f(x) = x(x-1)(x-2)(3-x)$. You are told that $\int_0^3 f(x) dx = \frac{9}{10}$ and $\int_1^2 f(x) dx = -\frac{11}{30}$. Without integrating further, find the total area between the curve and the x -axis for $0 \leq x \leq 3$.

Solution. The roots are $0, 1, 2, 3$. Test the sign: for $0 < x < 1$ the factors are $(+)(-)(-)(+) = +$; for $1 < x < 2$ they are $(+)(+)(-)(+) = -$; for $2 < x < 3$, $+$. So the curve is below the axis only on $(1, 2)$, where the integral $-\frac{11}{30}$ is minus an area. The signed integral over $[0, 3]$ is $\frac{9}{10}$, so the two positive pieces total $\frac{9}{10} + \frac{11}{30} = \frac{38}{30}$. Adding the negative piece's size gives

$$\text{total area} = \frac{38}{30} + \frac{11}{30} = \frac{49}{30}.$$

Examiner's eye. Equivalently: total area $= \frac{9}{10} + 2 \cdot \frac{11}{30}$. Check the reasoning with a symmetric picture: this quartic is symmetric about $x = \frac{3}{2}$, so the first and third pieces are equal, each $\frac{19}{30}$.

Equal areas. If a region above the axis and a region below the axis have the same area, their signed integrals cancel, so the integral over the *combined* interval is zero. This turns "find the parameter that makes two areas equal" into the single equation $\int_a^b f = 0$.



Worked example: regions of equal area

Question. The curve $y = (x - 2)(x - q)$, where $q > 2$, is shown. The region R lies between the curve, the y -axis and the x -axis for $0 \leq x \leq 2$; the region S lies between the curve and the x -axis for $2 \leq x \leq q$. Find q so that R and S have equal area.

Solution. R is above the axis (the value at $x = 0$ is $2q > 0$) and S is below it. Equal areas means $\int_0^2 f = -\int_2^q f$, i.e. $\int_0^q f(x) dx = 0$. Expand $f(x) = x^2 - (q + 2)x + 2q$:

$$\int_0^q f dx = \frac{q^3}{3} - \frac{(q+2)q^2}{2} + 2q^2 = q^2 \left(1 - \frac{q}{6}\right).$$

This is zero for $q = 6$ (the root $q = 0$ is not allowed since $q > 2$). Check: $\int_0^2 (x-2)(x-6) dx = \frac{32}{3}$ and $\int_2^6 (x-2)(x-6) dx = -\frac{32}{3}$.

Examiner's eye. The single equation $\int_0^q f = 0$ is much better than computing both areas separately. Notice the check by the values $\frac{32}{3}$ and $-\frac{32}{3}$: if they did not match, an arithmetic slip is hiding somewhere.

The area between two curves

The area enclosed between two curves is $\int (\text{upper} - \text{lower}) dx$ between their intersection points. The routine: (1) set the equations equal and solve to find the intersection x -values (these are your limits); (2) *sketch*, or test a value, to see which curve is on top between them; (3) integrate the difference. If the curves cross more than twice there are several regions and the "upper" curve changes, so do each region separately.

Symmetry halves the work. If both curves are odd, as in a line through the origin and an odd power, the picture is point-symmetric about the origin and the two enclosed regions are congruent. Compute one and double it, and then there are only positive limits to evaluate.

Strategy: the parabola and the chord

If a parabola $y = ax^2 + bx + c$ is cut by a straight line at $x = \alpha$ and $x = \beta$ ($\alpha < \beta$), the area between them is $\frac{|a|(\beta - \alpha)^3}{6}$, because the difference is $a(x - \alpha)(x - \beta)$ and $\int_\alpha^\beta (x - \alpha)(x - \beta) dx = -\frac{(\beta - \alpha)^3}{6}$. For example $y = x^2 - 3x$ and $y = x$ meet where $x^2 - 4x = 0$, at $x = 0, 4$, so the enclosed area is $\frac{4^3}{6} = \frac{32}{3}$. It saves a whole integration, but check the roots first.

Worked example: find the gradient from the area

Question. The line $y = mx$ (with $m > 0$) and the curve $y = x^5$ enclose two finite regions whose total area is 18. Find m .

Solution. They meet where $x^5 = mx$, i.e. $x(x^4 - m) = 0$, so $x = 0$ or $x = \pm a$ where $a^4 = m$. Both graphs are odd, so the two regions are congruent (a half turn about the origin maps one to the other). For $0 < x < a$ the line is above the curve (test $x = \frac{a}{2}$), so

$$\text{total} = 2 \int_0^a (mx - x^5) dx = 2 \left[\frac{ma^2}{2} - \frac{a^6}{6} \right] = 2 \left[\frac{a^6}{2} - \frac{a^6}{6} \right] = \frac{2}{3}a^6,$$

using $m = a^4$. Setting $\frac{2}{3}a^6 = 18$ gives $a^6 = 27$, $a^2 = 3$, so $m = a^4 = 9$.

Examiner's eye. The substitution $m = a^4$ is the neat move: it removes m and leaves a single power of a . Always express the parameter via the intersection point, not the other way round.

Worked example: two curves and a parameter

Question. The curves $y = x^2$ and $y = p - x^2$, where $p > 0$, enclose a region of area $\frac{64}{3}$. Find p .

Solution. They meet where $2x^2 = p$, at $x = \pm s$ with $s = \sqrt{p/2} > 0$, so $p = 2s^2$. The second curve is above the first between them, so

$$\text{area} = \int_{-s}^s (p - 2x^2) dx = 2 \left[ps - \frac{2}{3}s^3 \right] = 2 \left[2s^3 - \frac{2}{3}s^3 \right] = \frac{8}{3}s^3.$$

Setting $\frac{8}{3}s^3 = \frac{64}{3}$ gives $s^3 = 8$, $s = 2$, and $p = 2s^2 = 8$.

Examiner's eye. Cross-check with the parabola-and-chord strategy: the difference $p - 2x^2$ has $|a| = 2$ and the roots are $\pm s$, so the area is $\frac{2(2s)^3}{6} = \frac{8s^3}{3}$. The same.

Symmetry, geometry and the modulus

Many TMUA integrals are *shapes in disguise*. If the graph of the integrand is made of straight lines or part of a circle, the integral is an area you already know: a triangle ($\frac{1}{2}bh$), a trapezium, a rectangle, a semicircle ($\frac{1}{2}\pi r^2$). For the modulus, split at the point where the expression changes sign. The graph of $y = |x - 1|$ is a V with its vertex at $(1, 0)$, and the integral over any interval is the area of one or two triangles.

Odd and even. A function is **even** if $f(-x) = f(x)$ (its graph is symmetric about the y -axis) and **odd** if $f(-x) = -f(x)$ (symmetric under a half turn about the origin). Then

$$\int_{-a}^a f = 2 \int_0^a f \quad (\text{even}), \quad \int_{-a}^a f = 0 \quad (\text{odd}).$$

Any function $f(|x|)$ is even, since it only sees the size of x . A power of x is even if the exponent is even, odd if it is odd; a sum splits into its even and odd parts.

Strategy: draw the shape, then use geometry

Before integrating anything that contains $|\cdot|$ or $\sqrt{r^2 - x^2}$, sketch it. A V gives triangles. $y = \sqrt{r^2 - x^2}$ is the upper half of the circle $x^2 + y^2 = r^2$. A region like $|x| + 2|y| \leq 4$ is a rhombus with vertices on the axes, whose area is half the product of its diagonals.

Worked example: area from a picture

Question. Evaluate (a) $\int_{-3}^5 |x - 1| dx$; (b) $\int_{-4}^4 \sqrt{16 - x^2} dx$; (c) the area of the region $|x| + 2|y| \leq 4$.

Solution. (a) The V has vertex $(1, 0)$. On $[-3, 1]$ it is a triangle of base 4 and height 4, area 8; on $[1, 5]$ another of base 4 and height 4, area 8. Total 16. (b) $y = \sqrt{16 - x^2}$ is the upper semicircle of radius 4, so the integral is $\frac{1}{2}\pi \cdot 4^2 = 8\pi$. (c) The boundary $|x| + 2|y| = 4$ has vertices $(\pm 4, 0)$ and $(0, \pm 2)$: a rhombus with diagonals 8 and 4, so the area is $\frac{1}{2} \cdot 8 \cdot 4 = 16$.

Examiner's eye. For (c), if you integrate: for each x the vertical slice has $|y| \leq \frac{4-|x|}{2}$, a width of $4 - |x|$, and $\int_{-4}^4 (4 - |x|) dx = 16$. Same answer, but the diagram is quicker.

Worked example: odd and even parts

Question. Evaluate (a) $\int_{-2}^2 (x^5 + 3x^2 - x + 4) dx$ and (b) $\int_{-3}^3 (x^3 + 2) |x| dx$.

Solution. (a) x^5 and $-x$ are odd and contribute 0. The even part is $3x^2 + 4$, so the integral is $2 \int_0^2 (3x^2 + 4) dx = 2[x^3 + 4x]_0^2 = 2(8 + 8) = 32$. (b) Expand: $x^3|x| + 2|x|$. Here $x^3|x|$ is odd (odd times even) and contributes 0, while $2|x|$ is even: $2 \int_{-3}^3 |x| dx = 2 \cdot 9 = 18$ (two triangles of area $\frac{9}{2}$).

Examiner's eye. Beware the converse: $\int_{-a}^a f = 0$ for *one* value of a does not make f odd. For example $f(x) = x^2 - 1$

has $\int_{-\sqrt{3}}^{\sqrt{3}} f = 0$ and is even. (If the integral is zero for *every* a , then f is odd.)

Shifts, stretches and variable limits

Shifts. Moving the graph does not change an area: $\int_a^b f(x) dx = \int_{a-1}^{b-1} f(x+1) dx$, because $y = f(x+1)$ is the same graph one unit to the left. Look at the *limits* to see which part of f you are using: if x runs over $[1, 3]$ and the integrand is $f(x-1)$, then the argument of f runs over $[0, 2]$.

Stretches. $y = f(2x)$ is $y = f(x)$ squeezed horizontally by factor 2, so the area under it is halved: $\int_0^1 f(2x) dx = \frac{1}{2} \int_0^2 f(x) dx$. Check by substituting $u = 2x$: the limits double and the dx becomes $\frac{1}{2} du$.

Sums of unit integrals. Adjacent intervals join: $\int_0^1 f + \int_1^2 f + \dots + \int_{n-1}^n f = \int_0^n f$. A sum like $f(x) + f(x+1) + f(x+2)$ integrated over $[0, 1]$ is exactly this (each shifted piece lands on the next unit interval).

Worked example: using given values

Question. A continuous function f has $\int_0^2 f(x) dx = 5$ and $\int_0^4 f(x) dx = 12$. Find (a) $\int_1^3 f(x-1) dx$; (b) $\int_0^2 f(2x) dx$; (c) $\int_2^4 f(x) dx$.

Solution. (a) The argument $x-1$ runs over $[0, 2]$ as x runs over $[1, 3]$, so the integral equals $\int_0^2 f = 5$. (b) Halve the width: $\int_0^2 f(2x) dx = \frac{1}{2} \int_0^4 f(x) dx = 6$. (c) Subtract: $\int_2^4 f = \int_0^4 f - \int_0^2 f = 12 - 5 = 7$.

Examiner's eye. The function is not given. To make sure the rules are right, test them on a simple function that has the same data: $f(x) = \frac{x}{2} + 2$ has $\int_0^2 = 5$ and $\int_0^4 = 12$, and all three answers come out as above.

Worked example: adjacent integrals collapse

Question. Let $f(x) = |x-2|$. Evaluate $\int_0^1 [f(x) + f(x+1) + f(x+2) + f(x+3)] dx$.

Solution. The four terms integrate over $[0, 1]$, $[1, 2]$, $[2, 3]$, $[3, 4]$ respectively (replace $x+j$ by u), so the sum is $\int_0^4 |u-2| du$. This is two triangles, each of base 2 and height 2: $2 + 2 = 4$.

Examiner's eye. Writing four separate integrals of $|x+j-2|$ would take minutes; recognising a single integral over $[0, 4]$ takes seconds. Whenever you see the same function with $x, x+1, x+2, \dots$ think *shifted copies of adjacent intervals*.

Variable limits. If $F(x) = \int_a^x g(t) dt$ then $F'(x) = g(x)$: differentiating undoes integrating. When both limits depend on x , write $F(x) = G(\text{upper}) - G(\text{lower})$ where $G' = g$, so

$$\frac{d}{dx} \int_{u(x)}^{v(x)} g(t) dt = g(v(x))v'(x) - g(u(x))u'(x).$$

When the integrand is simple the safest route is to *integrate first and then differentiate*. A sequence defined by an integral, such as $u_n = \int_n^{n+1} \dots$, is handled the same way: evaluate, then use sequence and series skills.

Worked example: variable limits and an integral sequence

Question. (a) Let $F(x) = \int_x^{2x} (t^2 + 1) dt$. Find $F'(x)$ and $F'(1)$. (b) Let $u_n = \int_n^{n+1} (2x-1) dx$. Find u_n and $\sum_{n=1}^{20} u_n$.

Solution. (a) Integrate first: $F(x) = \left[\frac{t^3}{3} + t \right]_x^{2x} = \frac{8x^3}{3} + 2x - \frac{x^3}{3} - x = \frac{7x^3}{3} + x$. So $F'(x) = 7x^2 + 1$ and $F'(1) = 8$. (Via the rule: $F'(x) = 2((2x)^2 + 1) - (x^2 + 1) = 7x^2 + 1$.) (b) $u_n = \int_n^{n+1} (2x-1) dx = \left[x^2 - x \right]_n^{n+1} = ((n+1)^2 - (n+1)) - (n^2 - n) = 2n$. The sum is $2(1 + 2 + \dots + 20) = 2 \cdot 210 = 420$. Check by joining the integrals: $\sum u_n = \int_1^{21} (2x-1) dx = \left[x^2 - x \right]_1^{21} = 420 - 0 = 420$.

Examiner's eye. Two different routes to 420: summing the terms and joining the integrals. The agreement is your safety net.

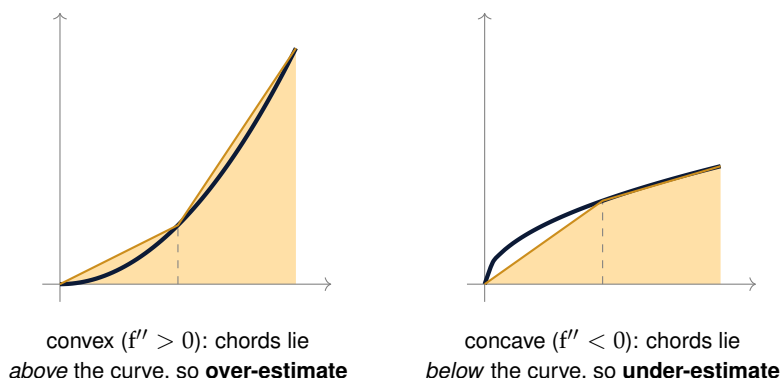
The trapezium rule

To estimate $\int_a^b f(x) dx$ with n equal strips of width $h = \frac{b-a}{n}$, replace the curve by chords joining neighbouring points (x_i, y_i) and add the trapezia:

$$\int_a^b f dx \approx \frac{h}{2} [y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n].$$

Every interior ordinate counts twice (it is the edge of two trapezia), the two end ordinates once.

Over or under? Each trapezium lies above the curve exactly where the chord is above the curve, that is where the curve is *convex*.



So for a function that is convex ($f'' > 0$) across the whole interval the rule **over-estimates**, and for one that is concave ($f'' < 0$) it **under-estimates**. If the curve changes shape inside the interval you cannot tell in general. The rule is *linear*: the estimate for $c f + d g$ is c times the estimate for f plus d times that for g , which is why a minus sign turns an over-estimate into an under-estimate.

Worked example: compute and compare

Question. Use the trapezium rule with 4 strips to estimate $\int_0^2 x^3 dx$. Is your estimate larger or smaller than the exact value? Explain without finding the exact value first.

Solution. Here $h = \frac{1}{2}$ and the ordinates are $0, \frac{1}{8}, 1, \frac{27}{8}, 8$. The estimate is

$$\frac{1/2}{2} \left[0 + 2\left(\frac{1}{8} + 1 + \frac{27}{8}\right) + 8 \right] = \frac{1}{4} [9 + 8] = \frac{17}{4}.$$

On $[0, 2]$ we have $f''(x) = 6x \geq 0$, so the curve is convex and the rule over-estimates. Check: the exact value is $\frac{2^4}{4} = 4 < \frac{17}{4}$.

Examiner's eye. The convexity argument costs five seconds and does not need the exact integral at all. In the exam, when the question asks only "over or under?", do not compute the estimate.

Worked example: transformed integrands

Question. The trapezium rule with 4 strips is used to estimate each of $A = \int_0^4 \sqrt{x} dx$, $B = \int_0^4 (6 - \sqrt{x}) dx$, $C = \int_{-4}^0 \sqrt{-x} dx$ and $D = \int_1^5 \sqrt{x-1} dx$. For which is the estimate an under-estimate?

Solution. $y = \sqrt{x}$ is concave on $[0, 4]$ ($f'' = -\frac{1}{4}x^{-3/2} < 0$), so A is **under-estimated**. B : the integrand $6 - \sqrt{x}$ is convex (minus a concave function), so B is **over-estimated**. C : $y = \sqrt{-x}$ is the mirror image of $\sqrt{x}$ in the y -axis, and a reflection does not change concavity, so C is under-estimated. D : $y = \sqrt{x-1}$ is $\sqrt{x}$ shifted one unit right, which also preserves concavity (and the strips are the same shapes), so D is under-estimated. The answer is A, C and D .

Examiner's eye. Change the integrand by a shift or reflection and the shape (convex or concave) travels with it. Change it by a minus sign or by subtracting from a constant and the shape *flips*.

Common mistakes

- Forgetting to double the interior ordinates, or doubling the end ordinates.
- Using the over/under rule when the curve is convex on part of the interval and concave on the rest.
- Saying a convex curve gives an under-estimate. Picture the chord: if it is above the curve, the trapezium has too much area.

Habits that win marks on integration questions

- **Sketch first.** A rough graph shows the sign of the function, the intersection points, which curve is on top and any symmetry, before a single integral is written.
- **Signed is not total.** If the curve crosses the axis between the limits, either split at the roots or use $\text{total} = \text{signed} + 2 \times (\text{the part below})$.
- **Look for a shape.** Triangles, semicircles, rhombuses: a diagram often replaces the integral completely.
- **Use what you are given.** If the question supplies integrals over some intervals, the answer is an addition or subtraction of them, perhaps after a shift or a halving of the width.
- **Put the parameter in terms of the intersection point.** For $y = mx$ and $y = x^5$, say $a^4 = m$; the algebra collapses.
- **Over or under?** Picture the chord. Chord above the curve (convex) means over-estimate. Check by one tiny strip if unsure.

Check yourself

Try these without looking back, then compare with the answers on the next page.

1. Evaluate $\int_1^4 \left(3\sqrt{x} - \frac{2}{\sqrt{x}} \right) dx$.
2. Find the area enclosed by $y = x^2$ and $y = x + 2$.
3. Find the total area between $y = x^3 - x$ and the x -axis for $-1 \leq x \leq 1$.
4. Evaluate $\int_{-2}^2 (x^3 + x^2 + 1) dx$.
5. Given $\int_0^4 f(x) dx = 10$, find $\int_0^2 f(2x) dx$.
6. The trapezium rule with 1 strip is used for $\int_0^4 \sqrt{x} dx$. Find the estimate and say whether it is an over- or under-estimate.
7. Find $\frac{d}{dx} \int_1^x (t^3 + t) dt$ and evaluate it at $x = 2$.
8. Find the area of the region $|x| + |y| \leq 3$.

Answers to the self-check

1. Antiderivative $2x^{3/2} - 4x^{1/2}$. At 4: $16 - 8 = 8$. At 1: $2 - 4 = -2$. Answer $8 - (-2) = 10$.
2. They meet where $x^2 - x - 2 = 0$, at $x = -1, 2$. Parabola-and-chord: $\frac{1 \cdot 3^3}{6} = \frac{9}{2}$.
3. Odd function: on $[0, 1]$ the area is $\int_0^1 (x - x^3) dx = \frac{1}{4}$, and the same on $[-1, 0]$. Total $\frac{1}{2}$, whereas the signed integral is 0.
4. The odd term x^3 gives 0. The rest is even: $2 \int_0^2 (x^2 + 1) dx = 2 \left(\frac{8}{3} + 2 \right) = \frac{28}{3}$.
5. Halve: the substitution $u = 2x$ gives $\frac{1}{2} \int_0^4 f(u) du = 5$.
6. $\frac{4}{2}(0 + 2) = 4$. The curve is concave, so it is an under-estimate; the exact value is $\frac{16}{3} > 4$.
7. The derivative of the integral is the integrand: $x^3 + x$, which is 10 at $x = 2$.
8. A square with vertices $(\pm 3, 0), (0, \pm 3)$, diagonals 6 and 6: area $\frac{1}{2} \cdot 6 \cdot 6 = 18$.

Next step

Now do the matching topic practice set: 20 questions in TMUA style, with worked solutions. Mark it, then redo the ones you missed a week later without looking at the solutions.